

Asset Pricing with Sustainability Concerns

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Abstract

We derive equilibrium prices and allocations in static and dynamic equilibrium models with concerns for sustainability. We show sustainability preferences lead to a breakdown of the Fisher separation between ownership and control, and the social suboptimality of the decentralized equilibrium even if markets are (effectively) complete and there are no externalities. Policy tools such as taxation, market-based solutions such as sustainability claims, and corporate governance tools such as monitoring boards can be used to implement social optimality. We document the impact of sustainability concerns on prices, volatility, risk premia and the interest rate. With CES preferences, the interest rate, market price of risk, equity premium and market volatility can become more sensitive to productivity shocks and exhibit greater variations in sensitivity across the productivity spectrum when a market for sustainability claims is introduced. The Green premium is mostly negative except under high substitutability and low productivity.

Keywords: Sustainability, Fisher separation, ownership, control, social optimum, taxes, sustainability claims, governance, pricing, Green premium, interest rate, volatility.

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1 Introduction

Sustainability has become a major concern for a broad spectrum of economic agents, including consumers, investors, managers and policy-makers. Sustainability is critical for survival, be it economic or physical. It conditions the use of resources and determines their availability across time and over the long run. In this paper, we examine the impact of sustainability concerns on equilibrium allocations and prices.

To this end, we consider various production economies, static and dynamic, with a composite consumption good and a sustainability index. The latter can be interpreted as an energy mix, e.g., Green energy versus Brown energy, as a labor mix, e.g., skilled versus unskilled or diverse versus homogeneous workforce, as a production process mix, e.g., polluting versus non-polluting technology, or as an organizational mix, e.g., strong versus weak governance. On the production side, the profit of the firm depends on the level of sustainability, and the decision-maker optimizes over it. On the consumption side, the agent's preferences depend on both the composite good and the sustainability index. The latter has the characteristics of a public good and consequently preferences have features of pro-social preferences. In such settings, we solve for the competitive (decentralized) equilibrium and compare it to the socially optimal (centralized) allocation.

Our contribution is fivefold. First, we show that the Fisher separation theorem (Fisher (1930)) which establishes the separation between ownership and control fail when investors care about sustainability, even if the financial market is complete or effectively complete and there are no externalities. This fundamental principle ensures that firms can optimize over production by maximizing the present value of future dividends taking prices as given, and that doing so is in the best interest of consumers irrespective of their preferences. The reason behind the failure of separation in the presence of sustainability concerns is because the agent cares about the production plan of the firm. Optimizing production by maximizing present value does not ensure alignment with the preferences of the agent. The competitive solution is therefore socially suboptimal, in fact the competitive sustainability index is strictly less than the socially optimal index, welfare is lower, and the standard present value rule fails as a managerial goal. The failure of the First Welfare Theorem in an economy with complete markets but pro-social preferences is a novel and conceptually significant result. Because the competitive equilibrium is no longer Pareto-efficient in this setting, there is clear scope for regulatory interventions or market-based mechanisms capable of improving upon the decentralized allocation.

Second, we examine solutions to the mismatch between the agent's preferences and the producer's incentives. We show that a properly designed tax can align the incentives of the producer and the consumer. Alternatively, creating a market for sustainability certificates can achieve the same objective. In both cases, the optimal policy of the firm ensures that the competitive sustainability choice is the same as the socially optimal choice, thus ensuring that the consumer preferences are implemented. The firm's valuation satisfies the present value rule under the solution adopted, and it deviates from its valuation in the absence of economic structure modifications. Conditions for lower/greater bond and stock values depend on the degree of substitution between the com-

posite good and the sustainability index in the agent’s preferences and on the structure of the profit function. The difference between the two solutions is that policy-intervention via taxes is information intensive, as it requires the regulator to know preferences and production functions. The market-based solution in contrast only requires knowledge of the certificates price as it aggregates all the relevant information for decentralized decision-making. The demand for certificates is determined by the agent’s preferences, and the instrument functions as a transfer mechanism that—consistent with the Second Welfare Theorem—enables the attainment of the socially optimal allocation. The certificates price operates as a decentralized information-aggregation and coordination signal, echoing Hayek (1945)’s insight that markets transmit dispersed knowledge more effectively than centralized regulatory tools such as taxes.

Third, we examine the implications of sustainability in a dynamic setting with a certificate market. We show the market price of risk, in addition to the usual component tied to consumption risk, is augmented by a premium associated with sustainability risk. The size of this premium is determined by the elasticity of the marginal utility of consumption with respect to sustainability and the sensitivity of sustainability to economic shocks. It is positive if the composite good and the sustainability index are substitutes in the agent’s preferences and if the exposure of sustainability to shocks is positive. As for the interest rate, it inherits components tied to the drift and volatility of sustainability and its covariance with aggregate consumption. Under CES preferences, the interest rate, price of risk and return volatility become more sensitive to productivity shocks across a range of elasticity values upon introduction of a certificates market. Nevertheless, while the volatility of equilibrium variables can increase, welfare also does.

Fourth, we identify key properties of Green premia—defined as the differences in equilibrium quantities between the socially optimal allocation and the decentralized competitive equilibrium associated with the transition to sustainability in the CES-preferences model. We show that (i) the Green premium in the interest rate is predominantly negative across the productivity spectrum except under complementarity at sufficiently high productivity; (ii) the Green premia in the price of risk and the expected market return are generally negative except when the elasticity of substitution exceeds one and productivity is sufficiently low; (iii) the Green volatility premium may be positive or negative depending on whether the composite good and sustainability are complements or substitutes; (iv) Green premia vary substantially over the productivity range; (v) their sensitivities with respect to productivity also vary markedly; and (vi) these premia can be large and exhibit sign reversals as productivity changes. Properties (iv)–(vi) are governed by the elasticity of substitution between the composite good and sustainability. Taken together, these results underscore that the magnitude and direction of Green premia are highly sensitive to underlying technological and preference parameters, making their empirical characterization both essential and nontrivial.

Last, we examine the socially optimal allocation of consumption and sustainability in an economy with heterogeneous consumers. Consumer heterogeneity generates additional challenges for attaining efficiency, since sustainability exhibits the characteristics of a public good. In particular, the firm supplies a fixed and uniform quantity of sustainability, while consumers’ willingness to pay

for this quantity differs. This asymmetry creates incentives for free-riding, which prevents the decentralized equilibrium from achieving the social optimum. We demonstrate that efficiency can be restored through a mechanism that combines individualized transfer payments with institutional enforcement. Specifically, we propose the formation of a shareholder consortium responsible for administering consumer-specific transfers. This arrangement preserves profit maximization as the firm’s objective—total dividends consist of both production returns and aggregate transfer payments—while simultaneously ensuring that the allocation of the consumption good satisfies social optimality. Furthermore, we show that such a mechanism can be operationalized through financial innovation by designing sustainability-linked assets that securitize the cash flows of individualized transfers.

The paper relates to three branches of the literature. First, it contributes to a long-standing stream of research on the separation of ownership and control. The seminal contribution of Fisher (1930) establishes the firm need not care about investor preferences in its decision-making in a certainty world, and the reverse. It shows value maximization provides a universal criterion ensuring investors are content with the firm’s choices and can focus on the selection of an optimal consumption-investment plan based on their preferences. The separation principle holds under uncertainty provided financial markets are complete; see, e.g., Debreu (1959), Arrow (1964) and Hirshleifer (1965). But it typically fails in the presence of frictions and agent diversity, as noted by Ekern and Wilson (1974), Radner (1974), Stiglitz (1974), Hart (1979), Grossman and Stiglitz (1980) and Smith (1996) for the case of incomplete markets, and Jensen and Meckling (1976), Myers and Majluf (1984) and Dewatripont and Tirole (1994) for the case of agency frictions. In such environments agents generally do not agree on valuation, which precludes the existence of a universal decision criterion and delegation of management to any particular individual. Our paper extends these insights to the case of sustainability preferences. We show that separation fails even if the market is complete or effectively complete, hence in the absence of financial or agency frictions. We also explore the design of remedies in the form of taxation and markets for sustainability certificates.

Second, it relates to more recent studies seeking to examine the economic role of sustainability in corporate decision-making. Heal (2005) describes channels through which Corporate Social Responsibility (CSR) programs aligning social and private incentives, hence leading to violations of profit maximization, can contribute to the long-term financial performance of firms. Benabou and Tirole (2010) argue that investors and insiders social preferences, e.g., environmental or ethical concerns, can influence firm decision-making through CSR and lead to widely different outcomes depending on the nature of the socially responsible behavior. In practice, investors seeking to affect the behavior of firms have pursued two main strategies, often referred to as Exit and Voice. Broadly-speaking Exit corresponds to exclusionary or divestment practices, and it is examined in Heinkel et al. (2001) and Landier and Lovo (2025). Voice encompasses various forms of shareholder activism and is dealt with in Chowdry et al. (2019), Morgan and Tumlinson (2019) and Oehmke and Opp (2025). In particular, Oehmke and Opp (2025) shows that a socially responsible fund

(SRF), with a broad mandate based on the reduction in social costs achieved by investing in a firm, can impact its technological choices. To achieve this outcome, the firm issues Green bonds bought by the SRF at a premium and regular bonds bought by profit-minded investors. Our analysis similarly leads to the socially optimal firm choice but, does not rely on the existence of a class of investors with a special mandate. Investors can differ in their preferences for sustainability in an arbitrary manner and one type of security traded in a competitive market is sufficient to achieve the socially optimal outcome. Our model further identifies the impact on the equilibrium interest rate, market risk premium and volatility of the market return.

Last, the paper also connects with a literature on public goods. Samuelson (1954) establishes foundations by identifying an efficiency condition for the production of a public good. Implementation can be carried out based on the personalized-pricing procedure proposed by Lindahl (1958). Subsequent contributions by Vickery (1961), Clarke (1971) and Groves (1973) address the potential for misrepresentation of preferences. They design truth-telling mechanisms that eliminate incentives for misreporting. The mechanism with individual contributions we propose for multi-agent economies draws on Lindahl's equilibrium. It differs to the extent that implementation can be carried out through a combination of procedures including a market-based solution involving sustainability claims. While the mechanism is not information-proof it circumvents incentives to renege and free-ride.

Section 2 presents a classic two-period model of the financial market to show the breakdown of the Fisher separation theorem in the presence of sustainability concerns and examine possible remedies. Section 3 extends the analysis to a dynamic asset pricing setting with a representative agent and studies the behavior of the interest rate, market price of risk and market volatility. Section 4 considers heterogeneous agents. Conclusions follow.

2 Asset Pricing in a 2-Period Model

2.1 The Decentralized Equilibrium

Consider a two-period setting with a bond and a stock market. The set of times is $\mathcal{T} = \{0, 1\}$. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be the underlying probability space where Ω is the set of states of nature with generic element $\omega \in \Omega$, $\mathcal{F}$ is the sigma-algebra of (eventually) observable events, and $\mathbb{P}$ is a probability measure on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The stock pays $D_0(s_0)$ at time $t = 0$, $D_1(s_1) \equiv D(s_1, \omega)$ at time $t = 1$ and is in unit supply. The bond pays 1 at time $t = 1$ and is in null supply.

The consumption space is $\mathcal{C} \equiv \mathbb{R}_+$. The sustainability space is a compact set $\mathcal{K} \subset \mathbb{R}_+$. The agent has utility function defined over the space $\mathcal{C} \times \mathcal{K}^2$ by

$$\mathcal{U}(c_0, s_0, s_1) = u(c_0, s_0) + a\mathbb{E}[v(c_1, s_1)] \quad (1)$$

where a is a subjective discount factor, c_0 is date 0 consumption, c_1 is date 1 consumption, and (s_0, s_1) are the sustainability indices at times $\{0, 1\}$. The functions u and v are twice continuously

differentiable, increasing, concave and satisfy the Inada conditions with respect to the first argument. The agent is endowed with the asset supply at the outset. Her budget constraint at the initial date is $c_0 + \pi_s P_0 + \pi_b B_0 \leq D_0 + P_0$ and terminal consumption is $c_1 = \pi_s D_1(s_1) + \pi_b$ where P_0 is the initial stock price, B_0 the bond price, π_s the number of shares of the stock bought/sold, and π_b the number of bonds bought/sold. Time 1 prices are $P_1 = D_1(s_1)$ and $B_1 = 1$.

Maximizing utility with respect to (c_0, π_s, π_b) subject to the budget constraint leads to the first order conditions

$$u_1(c_0, s_0) = y \quad (2)$$

$$\mathbb{E}[av_1(c_1, s_1)D_1(s_1)] = yP_0 \quad (3)$$

$$E[av_1(c_1, s_1)] = yB_0 \quad (4)$$

$$c_0 + \pi_s P_0 + \pi_b B_0 = D_0 + P_0 \quad (5)$$

$$c_1 = \pi_s D_1(s_1) + \pi_b. \quad (6)$$

Concavity of the utility functions ensures the second order conditions are satisfied. Since marginal utilities are invertible with respect to their first arguments and satisfy Inada conditions, a unique solution (c_0, π_s, π_b) exists.

The producer maximizes production with respect to the sustainability index $s \in \mathcal{K}$. We assume the production function is twice continuously differentiable, strictly concave with (negative) infinite slope at the (upper) lower boundary of $\mathcal{K}$.¹ First order conditions are

$$D'_0(s_0) = 0; \quad D'_1(s_1) = 0 \quad (7)$$

where $D'_1(s_1)$ is the derivative relative to the sustainability argument of the production function. The properties of the production function ensure there is a unique solution denoted by (s_0^*, s_1^*) . The policy s_1^* depends on the state ω .

With these preliminaries, we obtain the following result.

Proposition 2.1. *Equilibrium in the decentralized economy exists. Equilibrium consumption is $c_0^* = D_0(s_0^*)$ and equilibrium investment $\pi_s^* = 1$ and $\pi_b^* = 0$. The equilibrium (ex-dividend) stock price is given by*

$$P_0 = \mathbb{E} \left[\frac{av_1(D_1(s_1^*), s_1^*)}{u_1(D_0(s_0^*), s_0^*)} D_1(s_1^*, \omega) \right] \quad (8)$$

$$B_0 = \mathbb{E} \left[\frac{av_1(D_1(s_1^*), s_1^*)}{u_1(D_0(s_0^*), s_0^*)} \right] \quad (9)$$

where (s_0^*, s_1^*) is the profit-maximizing sustainability policy of the firm.

¹Increasing sustainability therefore raises (lowers) profits from operations below (above) some threshold. Examples of sustainability actions with these properties include investments linked to pollution control. Maintenance of a pollution-free environment promotes production, yet marginal net benefit falls and eventually becomes negative when investment becomes excessive.

2.2 The Central Planner Problem

Consider now the centralized allocation problem. We add the following assumption on the marginal rates of substitution between consumption and sustainability.

Assumption 2.2. *Let $m_0(c, s) \equiv u_2(c, s)/u_1(c, s)$ be the marginal rate of substitution at time $t = 0$ and $m_1(c, s) \equiv v_2(c, s)/v_1(c, s)$ at $t = 1$. Each of the maps*

$$\Psi_t(s) \equiv m_t(D_t(s), s) + D'_t(s); \quad t \in \mathcal{T} \quad (10)$$

has a unique zero, i.e., there exists a unique s_t^o such that $\Psi_t(s_t^o) = 0$.

The function $\Psi_t(s)$ is the marginal rate of substitution augmented by the marginal dividend at time $t \in \mathcal{T}$, evaluated at aggregate consumption $c_t = D_t(s)$. The condition is fairly weak. It is automatically satisfied if $m_t(D_t(s), s)$ is decreasing in s . It also allows for $\Psi_t(s)$ to have non-monotone behavior off the singleton s_t^o . The only requirement is the single crossing property at 0. Examples 2.5 and 2.12 display cases where the single crossing property holds under even weaker assumptions on production.

The central planner maximizes utility $\mathcal{U}(c, s_0, s_1)$ with respect to $(c, s_0, s_1) \in \mathcal{C} \times \mathcal{K}^2$ subject to the resource constraint. First order conditions are

$$u_1(c_0, s_0) = y \quad (11)$$

$$u_2(c_0, s_0) = -yD'_0(s_0) \quad (12)$$

$$av_1(D_1(s_1), s_1)D'_1(s_1) + av_2(D_1(s_1), s_1) = 0 \quad (13)$$

$$c_0 = D_0(s_0) \quad (14)$$

where y is the Lagrange multiplier for the resource constraint $c_0 = D_0(s_0)$. Under Assumption 2.2 a unique optimal solution (c^o, s_0^o, s_1^o) exists.

Proposition 2.3. *Suppose Assumption 2.2 holds. The social optimum policy is (c^o, s_0^o, s_1^o) where s_t^o is the unique solution of $\Psi_t(s_t) = 0$ for $t \in \mathcal{T}$ and $c_0^o = D_0(s_0^o)$. Socially optimal sustainability indices exceed optimal sustainability indices in the decentralized economy, $s_0^o > s_0^*$ and $s_1^o > s_1^*$, optimal consumption is lower $c_0^o < c_0^*$ and welfare is higher $\mathcal{U}^o > \mathcal{U}^*$. The Fisher separation between ownership and control fails in the decentralized economy.*

Proof of Proposition 2.3 Existence of unique optimal sustainability policies $s_t^o, t \in \mathcal{T}$ follows from Assumption 2.2. Optimal consumption $c_0^o = D_0(s_0^o)$ satisfies the resource constraint. It also follows from the optimality conditions that

$$D'_0(s_0^o) = -\frac{u_2(D_0(s_0^o), s_0^o)}{u_1(D_0(s_0^o), s_0^o)}; \quad D'_1(s_1^o) = -\frac{v_2(D_1(s_1^o), s_1^o)}{v_1(D_1(s_1^o), s_1^o)} \quad (15)$$

in the planning problem, whereas $D'_0(s_0^*) = D'_1(s_1^*) = 0$ in the decentralized problem. Since marginal utilities are positive and the production function is continuously differentiable and concave with

negative infinite derivative at the upper boundary of $\mathcal{K}$, it follows $s_0^o > s_0^*$ and $s_1^o > s_1^*$. As a result $c_t^o = D_t(s_t^o) < D_t(s_t^*) = c_t^*$. That decentralized utility is dominated follows from the fact that the competitive economy is a constrained version of planning economy. $\blacklozenge$

Remark 2.4. *The Fisher separation principle does not fail if $u_2 = v_2 = 0$. In this case, the agent does not care about sustainability implying value maximization aligns with wealth maximization, thus providing maximal budget flexibility.*

Example 2.5. *Suppose $u_t(c, s) = \log(c) + ks$ and $D_t(s) = A_t s(1-s)$ for $t \in \mathcal{T}$, where A_0 is constant and A_1 a random productivity coefficient. By definition $v(c, s) \equiv u_1(c, s)$. Optimal policies in the decentralized problem solve*

$$c_0^{-1} = y; \quad c_t = D_t(s_t); \quad A_t(1 - 2s_t) = 0 \quad (16)$$

and are given by $s_0^* = s_1^* = 1/2$ and $c_0^* = 1/4$.

Optimal policies in the social planning problem solve

$$c_0^{-1} = y; \quad c_t = D_t(s_t); \quad k + yA_0(1 - 2s_0) = 0; \quad \frac{1 - 2s_1}{s_1(1 - s_1)} = -k. \quad (17)$$

The solution is

$$\Delta = 4 + k^2 \quad (18)$$

$$s^o \equiv s_0^o = s_1^o = (k - 2 + \sqrt{\Delta})/2k \quad (19)$$

$$c_t^o = A_t s^o(1 - s^o), \quad t \in \mathcal{T} \quad (20)$$

$$u_t^o = \log(c_t^o) + ks^o, \quad t \in \mathcal{T}. \quad (21)$$

Optimal sustainability is time-independent. Optimal consumption reflects the productivity factor. The right panel of Figure 1 shows the socially optimal allocation dominates the decentralized equilibrium and entails greater sustainability. The left panel illustrates the Fisher separation between ownership and control for the pure consumption preferences in Remark 2.4.

2.3 Implementing the Socially Optimal Outcome

This section examines mechanisms, specifically sustainability taxes and certificates, that can be used to implement the socially optimal allocation in the decentralized economy.

2.3.1 Sustainability Taxes

The next result provides a welfare comparison across economies and shows how to design socially optimal taxes.

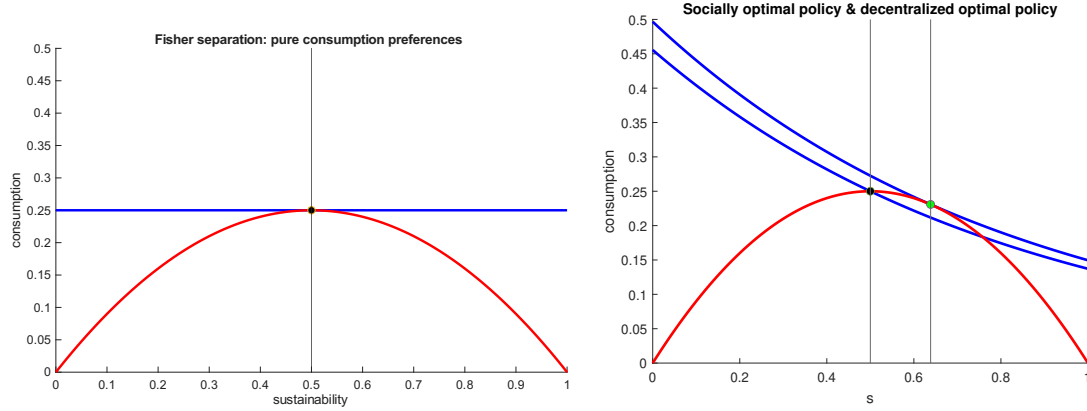


Figure 1: The plot shows the indifference curve (blue) and the production set (red) at time $t = 0$ in Example 2.5 with $A_0 = 1$, $k = 1.2$, for pure consumption preferences (left panel) and preferences for sustainability (right panel). The socially optimal policy in the right panel is marked in green. The decentralized optimum is in black. The decentralized solution is dominated.

Proposition 2.6. *Consumption is greater while sustainability and welfare are lower in the decentralized economy as compared to the centralized economy*

$$s_0^o > s_0^*; \quad s_1^o > s_1^* \quad (22)$$

$$c_0^o = D_0(s_0^o) < D_0(s_0^*) = c_0^*; \quad c_1^o = D_1(s_1^o) < D_1(s_1^*) = c_1^* \quad (23)$$

$$u^o \equiv u(c_0^o, s_0^o) > u(c_0^*, s_0^*) \equiv u^*; \quad v^o \equiv v(c_1^o, s_1^o) > v(c_1^*, s_1^*) \equiv v^*. \quad (24)$$

The socially optimal allocation $(c_t^o, s_t^o), t \in \{0, 1\}$ is attained in the decentralized economy if the regulator imposes a sustainability tax

$$\tau(s_t) = (D(s_t) - D(s_t^o))^+ + \epsilon 1_{\{s_t \neq s_t^o\}} \quad (25)$$

for $\epsilon > 0$, at time $t \in \mathcal{T}$. The associated profit of the firm and equilibrium consumption are $c_t^o = D_t(s_t^o), t \in \mathcal{T}$. The effective tax at the optimum is null $\tau(s_t^o) = 0, t \in \mathcal{T}$.

Proof of Proposition 2.6 Optimality of the social planning problem ensures $u^* < u^o, t$ and $v^* < v^o$. Optimality of dividend maximization ensures $D_t(s_t^*) > D_t(s_t^o), t \in \mathcal{T}$. That $s_t^* < s_t^o, t \in \mathcal{T}$ was shown previously. After-tax dividend maximization

$$\max_{s_t} (D_t(s_t) - \tau(s_t)) = D_t(s_t^o) - \epsilon \min_{s_t} 1_{\{s_t \neq s_t^o\}} = D_t(s_t^o)$$

implements the socially optimal dividend and the socially optimal sustainability index given by $s_t^o = \arg \max_{s_t} (D_t(s_t) - \tau(s_t))$ at each time $t \in \mathcal{T}$. At the optimum, the effective tax is $\tau(s_t^o) = 0$. ♦

The tax penalizes any deviation from the socially optimal sustainability level that raises operational profits to the fullest extent plus an epsilon. It also taxes the sustainability level $s_t^1 < s_t^o$

that reaches the same operational profit $D_t(s_t^1) = D_t(s_t^o)$ by an epsilon. This ensures the firm implements the socially optimal sustainability. Associated consumption and sustainability are also implemented. The effective tax is null because the deviation from the socially optimal dividend is null in the competitive equilibrium with taxation.

To provide perspective, suppose the tax does not depend on sustainability, i.e., $\tau(s) = \bar{\tau}D(s_t)$ for constant tax rate $\bar{\tau}$. This taxation scheme will diminish the dividend but does not alter the profit-maximizing sustainability choice, i.e., the firm chooses $s = s^*$. The consumer collects the tax proceeds $\bar{\tau}D(s^*)$ but receives less dividends $(1 - \bar{\tau})D(s^*)$. Since total income is unaffected, she chooses the consumption policy $c = c(s^*)$. The constant proportional tax therefore does not implement the socially optimal sustainability choice. If the tax rate depends on s , as in $\tilde{\tau}(s)$, then the optimal choice of the taxed firm solves $D'_t(\tilde{s}_t)(1 - \tilde{\tau}_t(\tilde{s}_t)) = D_t(\tilde{s}_t)\tilde{\tau}'(\tilde{s}_t)$ and optimal consumption is $\tilde{c}_t = D(\tilde{s}_t)$.

Although sustainability-contingent taxation can align the preferences of the consumer with the objective of the producer, it places a heavy informational burden on the regulator. Indeed, to ensure the producer selects the socially optimal sustainability level, the tax must depend on the preferences of the consumer. Incomplete information about preferences limits the implementation of this solution.

2.3.2 Financial Innovation: Sustainability Certificates

Another solution is to open a market for sustainability certificates which incentivize the producer to implement a desired sustainability allocation. Certificates are traded in this market, and they essentially function as transfers from the consumer to the producer. The firm's income from selling (supplying) s_t certificates is $p_t s_t$ and this choice generates a profit equal to $D_t(s) + p_t s_t$. The profit-maximizing sustainability choice is $\hat{s}_t(p_t) = \arg \max_s (D_t(s) + p_t s)$. Certificates are bought (demanded) by the consumer who pay $p_t s_t$. As she owns the firm, she receives the dividend $D_t(\hat{s}_t) + p_t \hat{s}_t$ and pays the certificates cost $p_t s_t$. She therefore solves $\max_{(c,s)} u_t(c, s) : c + p_t s_t \leq D_t(\hat{s}_t(p_t)) + p_t \hat{s}_t(p_t)$ at time $t \in \mathcal{T}$, where we use the compact notation $u_t \equiv u$ for $t = 0$ and $u_t \equiv v$ for $t = 1$.

Proposition 2.7. *For $t \in \mathcal{T}$, the profit maximizing choice of s_t in the presence of a market for sustainability certificates with price p_t is*

$$\hat{s}_t(p_t) = \arg \max_s (D_t(s) + p_t s) = [D'_t]^{-1}(-p_t). \quad (26)$$

Given p_t the consumer chooses

$$(\hat{c}_t, \hat{s}_t) = \arg \max_{c,s} \{u_t(c, s) : c + p_t s \leq D_t(\hat{s}_t(p_t)) + p_t \hat{s}_t(p_t)\}. \quad (27)$$

The equilibrium certificate price is

$$\hat{p}_t = -D'_t(s_t^o) \quad (28)$$

and it corresponds to the socially optimal price. Given the equilibrium certificate price, the consumer chooses $\hat{c}_t(\hat{p}_t) = c_t^o$ and $\hat{s}_t(\hat{p}_t) = s_t^o$, the socially optimal consumption and sustainability levels at time $t \in \mathcal{T}$.

Proof of Proposition 2.7 The profit maximizing sustainability choice of the producer satisfies $D'_t(\hat{s}_t) = -p_t$. The utility-maximizing choices (c, s) of the consumer satisfy

$$u_{1t}(c_t, s_t) = y; \quad u_{2t}(c_t, s_t) = yp_t; \quad c_t + p_t s_t = D(\hat{s}_t(p_t)) + p_t \hat{s}_t \quad (29)$$

where u_{1t} (resp. u_{2t}) is the marginal utility with respect to consumption (resp. sustainability) at time t . Equating supply and demand $s_t = \hat{s}_t$ shows the equilibrium price satisfies

$$p_t = \frac{u_{2t}(c_t, s_t)}{u_{1t}(c_t, s_t)} = -D'_t(s_t) \quad (30)$$

where $c_t = D_t(s_t)$. It follows the equilibrium price solves $\Psi_t(s) = 0$ where $\Psi_t(\cdot)$ is defined in (10). The solution is as stated in the proposition. ♦

Proposition 2.7 shows the socially optimal solution can be attained by introducing a market for sustainability certificates. In contrast to taxation, this market-based solution does not place any informational burden on the firm as the certificate price signals value. Profit maximization accounting for certificate revenues ensures the firm attains the socially optimal sustainability in the competitive equilibrium with certificate market.

2.4 Asset Prices in the Socially Optimal Equilibrium

We now focus on the socially optimal equilibrium in the economy with a certificate market. Discounted prices are martingales in this economy. Our first result examines their behavior in relation to sustainability. For clarity, we use the notation $v(c, s)$ (resp. $u(c, s)$) for utility at time $t = 1$ (resp. $t = 0$).

Corollary 2.8. *The bond and stock prices evaluated at shadow prices from the planner's problem are*

$$B_0^o = \mathbb{E} \left[\frac{av'(D_1(s_1^o(\omega)), \omega), s_1^o)}{u_1(D_0(s_0^o), s_0^o)} \right] \quad (31)$$

$$P_0^o = \mathbb{E} \left[\frac{av'(D_1(s_1^o), s_1^o)}{u_1(D_0(s_0^o), s_0^o)} D_1(s_1^o) \right]. \quad (32)$$

The discounted bond price $u_1 B_0^o$ is greater in the centralized economy. The discounted stock price $u_1 P_0^o$ is greater (smaller) in the centralized economy if the elasticity of substitution between consumption and sustainability is less (greater) than 1.

Proof of Corollary 2.8 First, recall that the elasticity of substitution is

$$\epsilon = \frac{d \log(r)}{d \log(v_2/v_1)} = \frac{v_1 v_2 / (rs)}{v_{21} v_1 - v_{11} v_2} \quad (33)$$

where $r = c/s$. Second, the total derivative of the marginal utility weighted payoff $v_1(D_1(s_1^o), s_1^o)D_1(s_1^o)$ is

$$(v_{11}(D_1(s_1^o), s_1^o)D_1'(s_1^o) + v_{12}(D_1(s_1^o), s_1^o))D_1(s_1^o) + v_1(D_1(s_1^o), s_1^o)D_1'(s_1^o)) \quad (34)$$

and it is positive (negative) if and only if (omitting arguments of functions)

$$-v_{11}v_2 + v_{12}v_1 - v_1v_2/D_1 = \frac{v_1v_2}{rs} \left(\frac{1}{\epsilon} - 1 \right) > 0; \quad (< 0). \quad (35)$$

The same condition without the last term applies for the bond price. If $\epsilon < 1$ ($\epsilon > 1$) the above condition is positive (negative), which establishes the result for the discounted stock price. For the discounted bond price the condition is always positive. ♦

Next, we examine the marginal effect of sustainability s on stock and bond prices in the two economies. These marginal effects capture the variations in Green premia, i.e., sustainability premia, in the equilibria under considerations.

In the decentralized economy, the marginal impact of an increase in s_1^* on equilibrium prices is

2

$$\frac{\partial}{\partial s_1} B_0^* = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \left(\mathbb{E} \left[\frac{\beta v_1(D_1(s_1^* + \epsilon), s_1^* + \epsilon)}{u_1(D_0(s_0^*), s_0^*)} \right] - \mathbb{E} \left[\frac{\beta v_1(D_1(s_1^*), s_1^*)}{u_1(D_0(s_0^*), s_0^*)} \right] \right) \quad (36)$$

for the bond price and, with $\tilde{v}_1(D, s) \equiv v_1(D, s)D$,

$$\frac{\partial}{\partial s_1} P_0^* = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \left(\mathbb{E} \left[\frac{\beta \tilde{v}_1(D_1(s_1^* + \epsilon), s_1^* + \epsilon)}{u_1(D_0(s_0^*), s_0^*)} \right] - \mathbb{E} \left[\frac{\beta \tilde{v}_1(D_1(s_1^*), s_1^*)}{u_1(D_0(s_0^*), s_0^*)} \right] \right) \quad (37)$$

for the stock price. Since the effect through the first argument is null by optimality ($D_1'(s_1^*) = 0$), it follows that

$$\frac{\partial}{\partial s_1} B_0^* = \mathbb{E} \left[\frac{\beta v_{12}(D_1^*, s_1^*)}{u_1(D_0^*, s_0^*)} \right]; \quad \frac{\partial}{\partial s_1} P_0^* = \mathbb{E} \left[\frac{\beta v_{12}(D_1^*, s_1^*)D_1^*}{u_1(D_0^*, s_0^*)} \right]. \quad (38)$$

The next proposition establishes a necessary and sufficient condition for an increase in the Green premium.

Proposition 2.9. *In the decentralized equilibrium, the Green premia in stock and bond prices both increase at the margin if and only if (c_1^*, s_1^*) are utility complements (substitutes).*

²We assume $|v_{12}(D_1(\tilde{s}), \tilde{s})| \leq H$ for all $\tilde{s} \in (s_1^* - \epsilon, s_1^* + \epsilon)$ some $\epsilon > 0$ and some random variable H such that $\mathbb{E}[H] < \infty$, to ensure dominated convergence applies and the derivatives in (36)-(37) are well defined.

Proof of Proposition 2.9. If (c_1^*, s_1^*) are utility complements, $v_{12} \geq 0$. If they are utility substitutes, $v_{12} \leq 0$. The result then follows from (38). ♦

In the socially optimal equilibrium, the impact of a perturbation on the bond price decomposes as

$$\frac{\partial}{\partial s_1} B_0^o = \mathbb{E} \left[\frac{\beta v_{12}(D_1^o, s_1^o)}{u_1(D_0^o, s_0^o)} \right] + \mathbb{E} \left[\frac{\beta v_{11}(D_1^o, s_1^o) D_1'(s_1^o)}{u_1(D_0^o, s_0^o)} \right]. \quad (39)$$

Taking account of optimality $D_1'(s^o) = -v_2(D_1(s_1^o), s_1^o)/v_1(D_1(s_1^o), s_1^o) \leq 0$, we can state the following.

Proposition 2.10. *The Green premium in the bond price increases at the margin in the socially optimal equilibrium if (s_1^o, c_1^o) are utility complements.*

As for the ex-dividend stock price, with relative risk aversion coefficient $R_1(c, s) = -v_{11}(c, s)c/v_1(c, s)$ and the definition $R_1^o(s_1^o) \equiv R_1(c_1^o, s_1^o)$, we have

$$\frac{\partial}{\partial s_1} P_0^o = \mathbb{E} \left[\frac{\beta v_{12}(D_1^o, s_1^o) D_1(s_1^o)}{u_1(D_0^o, s_0^o)} \right] + \mathbb{E} \left[\frac{\beta v_{11}(D_1^o, s_1^o)}{u_1(D_0^o, s_0^o)} (1 - R_1^o(s_1^o)) D_1'(s_1^o) \right] \quad (40)$$

so that

Proposition 2.11. *The Green premium in the stock price increases at the margin in the socially optimal equilibrium if (c_1^o, s_1^o) are utility complements and if relative risk aversion $R_1^o(s_1^o) \geq 1$. It decreases at the margin if (c_1^o, s_1^o) are utility substitutes and $R_1^o(s_1^o) \leq 1$.*

The results of Propositions 2.10 and 2.11 imply that it is possible to have a simultaneous increase at the margin in the Green stock and bond premia. Complementarity and relative risk aversion greater than 1 suffice for that purpose. Complementarity and myopia, in particular, are sufficient. Equilibrium can also entail an increasing Green Stock premium and a decreasing Bond premium at the same time, and vice versa. The risk aversion effect must dominate the substitution/complementarity effect in the stock price for such patterns to emerge.

Example 2.12. *Consider an economy with $u(c, s) = \log c + \eta cs$ and production set as in Example 2.5 but with log-normally distributed productivity. In this setting s^o solves*

$$\eta A_t s_t^o (1 - s_t^o) = - \left(\frac{1}{A_t s_t^o (1 - s_t^o)} + \eta s_t^o \right) A_t (1 - 2s_t^o) \quad (41)$$

and $c_t^o = A_t s_t^o (1 - s_t^o)$ where $A_t = \kappa e^{\mu + \sigma x - (1/2)\sigma^2}$ for $t = 1$ with x a standard normal random variable and coefficients (κ, μ, σ) constants, and $A_t = 1$ for $t = 0$. The function Ψ has a single zero, ensuring s^o is unique for each realization of $x \in (-\infty, \infty)$. Figure 2 shows the discrepancy between the optimal sustainability levels in the original decentralized economy equilibrium and the socially optimal economy. Figure 3 displays the impact of the utility complementarity parameter $u_{12} = \eta$ on prices and the variation in prices (upper panels), and returns and the variation in returns (lower panels). In this example both prices (returns) increase (decrease) with η . The premium in

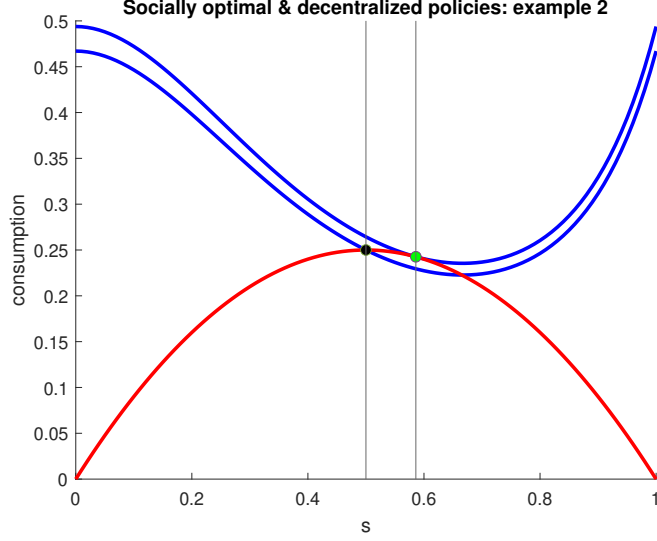


Figure 2: The plot shows the indifference curve (blue) and the production set (red) at time $t = 0$ in Example 2.12. The socially optimal policy (green) dominates the decentralized optimum (black). Parameter values are $k = 5, \beta = 0.01, \kappa = 2, \mu = 0.02, \sigma = 0.25, R = 2, \rho = 0.5, \alpha = 0.5$.

prices (returns) for switching to the social optimum is positive (negative). The premium in the bond (stock) price is increasing (increasing-decreasing) over the range of η examined. The premia in bond and stock returns are decreasing over the range of η .

3 Asset Pricing in a Dynamic Model

Consider now a dynamic production economy with infinite time horizon $\mathcal{T} = [0, \infty)$. The underlying probability space is $(\Omega, \mathcal{F}, \mathbb{P})$ and the information filtration $\mathbb{F} = \{\mathcal{F}_t : t \in \mathcal{T}\}$ is generated by a d -dimensional Brownian motion W . The firm pays a stochastic dividend process $D = \{D_t(s_t) : t \in \mathcal{T}\}$ where s is a sustainability process and the subscript t on D_t indicates a dependence on a stochastic factor, e.g., productivity. We assume the dividend and sustainability processes are non-negative, progressively measurable ($\mathcal{PM}$) and integrable ($\mathcal{I}$). The dividend space is denoted by $\mathcal{D}$ and the sustainability space by $\mathcal{S}$. Processes in $\mathcal{S}$ take values in a compact set $\mathcal{K}$. The stock of the firm is in unit supply and the stochastic stock price process is denoted by P . We consider both the economies with and without a market for sustainability certificates. If traded, the certificates price is a stochastic process denoted by p . In what follows, we focus on the economy with a certificates market. The goal is to derive the equilibrium prices (P, p) , the interest rate r and the market price of risk θ in this economy. Equilibrium in the model without the certificates market is obtained as a sub-case.

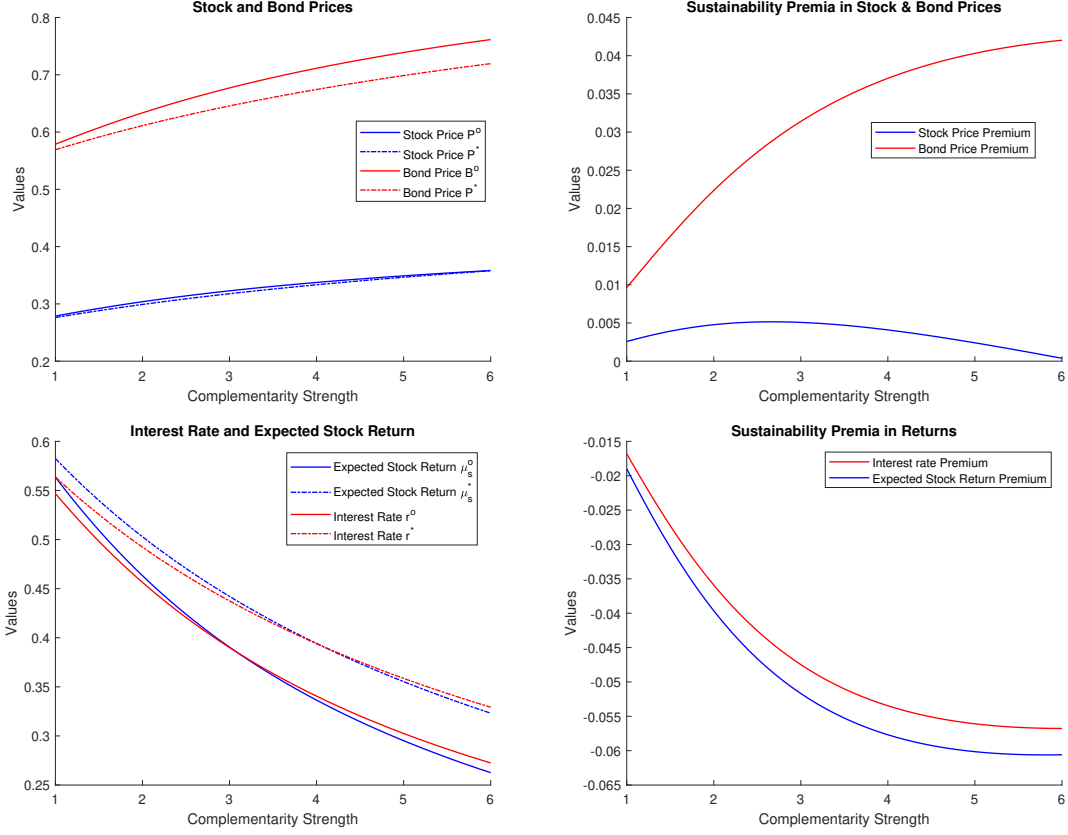


Figure 3: Panels show prices (upper left), variations in sustainability price premia (upper right), returns (lower left) and variations in sustainability premia in returns (lower right) at $t = 0$ in Example 2.12 when the degree of complementarity η varies from 1 to 6. Other parameters are $k = 10, \beta = 0.01, \kappa = 2, \mu = 0.02, \sigma = 0.25, R = 2, \rho = 0.5, \alpha = 0.5$.

3.1 Equilibrium with a Certificates Market

The consumption space is the set of non-negative, progressively measurable and integrable stochastic processes $\mathcal{C}$. Preferences are defined over $\mathcal{C} \times \mathcal{S}$ by the von Neumann-Morgenstern index

$$\mathcal{U}(c, s) = \mathbb{E} \left[\int_0^\infty a_v u(c_v, s_v) dv \right] \quad (42)$$

where the utility function u is twice continuously differentiable, increasing, concave and satisfies the Inada conditions with respect to the first argument. Here, a_t is a subjective discount factor with discount rate β , a progressively measurable and non-negative stochastic process. Optimization with respect to $(c, s) \in \mathcal{C} \times \mathcal{S}$ subject to the static budget constraint leads to the necessary conditions

$$u_1(c_t, s_t) = y \xi_t / a_t \quad (43)$$

$$u_2(c_t, s_t) = (y \xi_t / a_t) p_t \quad (44)$$

$$\mathbb{E} \left[\int_0^\infty \xi_t (c_t + p_t s_t) dt \right] = x \quad (45)$$

where $x = P_0$ is initial wealth and y is the Lagrange multiplier for the static budget constraint.

The firm maximizes the present value of dividends with respect to the sustainability input process $s \in \mathcal{S}$

$$\max_{s \in \mathcal{S}} \mathbb{E} \left[\int_0^\infty \xi_t (D_t(s_t) + p_t s_t) dt \right] \quad (46)$$

where the function $D_t(\cdot)$ is twice continuously differentiable, concave with negative infinite (positive infinite) slope at the upper (lower) boundary of the domain $\mathcal{K}$ for each $t \in [0, \infty)$. As a function of t , the production function is an adapted and non-negative stochastic process. Stochastic fluctuations in D represent, for instance, productivity shocks. Optimization gives the policy that solves $D'_t(s_t) + p_t s_t = 0$.

The counterpart of Assumption 2.2 in this dynamic model is next. The same remarks apply.

Assumption 3.1. *Let $m(c, s) \equiv u_2(c, s)/u_1(c, s)$ be the marginal rate of substitution. The stochastic map*

$$\Psi_t(s) \equiv m_t(D_t(s), s) + D'_t(s); \quad t \in [0, \infty) \quad (47)$$

has a unique zero, i.e., there exists a unique s_t^o such that $\Psi_t(s_t^o) = 0$.

Now define the preference parameters (ignoring arguments)

$$R^c = -\frac{u_{11}c}{u_1}; \quad R^s = -\frac{u_{22}s}{u_2}; \quad P^c = -\frac{u_{111}c}{u_{11}} \quad (48)$$

$$\mathcal{E}^{cs} = -\frac{u_{12}s}{u_1}; \quad \mathcal{E}^{cs} = -\frac{u_{112}s}{u_{11}}; \quad \mathcal{E}^{css} = -\frac{u_{122}s}{u_{12}} \quad (49)$$

where (c, s) stand for the arguments of the function. Note that (R^c, R^s) are conditional relative risk aversion measures with respect to consumption and sustainability, P^c is the conditional relative prudence coefficient with respect to consumption, $\mathcal{E}^{cs}$ is the elasticity of the marginal utility of consumption u_1 with respect to sustainability, and $(\mathcal{E}^{ccs}, \mathcal{E}^{css})$ are the elasticities of the second-order partial derivatives (u_{11}, u_{12}) with respect to sustainability.

Proposition 3.2. *Consider the economy with a market for sustainability certificates and assume Assumption 2.2 holds. A competitive rational expectations equilibrium exists. Equilibrium sustainability and consumption are $s_t^o = [\Psi_t]^{-1}(0)$ and $c_t^o = D_t(s_t^o)$ with respective dynamics*

$$\frac{ds_t^o}{s_t^o} = \mu_{st}^o dt + \sigma_{st}^o dW_t; \quad \frac{dc_t^o}{c_t^o} = \mu_{ct}^o dt + \sigma_{ct}^o dW_t. \quad (50)$$

The equilibrium state price density is $y\xi_t^o = a_t u_1(D_t(s_t^o), s_t^o)$ for $t \in [0, \infty)$. The equilibrium interest rate and market price of risk are

$$r_t^o = \beta_t + R_t^c \mu_{ct}^o - \frac{1}{2} R_t^c P_t^c (\sigma_{ct}^o)^2 + \mathcal{E}_t^{cs} \mu_{st}^o - \frac{1}{2} \mathcal{E}_t^{cs} \mathcal{E}_t^{css} (\sigma_{st}^o)^2 - R_t^c \mathcal{E}_t^{ccs} \sigma_{cst}^o \quad (51)$$

$$\theta_t^o = R_t^c (\sigma_{ct}^o)' + \mathcal{E}_t^{cs} (\sigma_{st}^o)' \quad (52)$$

where all preference coefficients are evaluated at the equilibrium allocation (c^o, s^o) and $\sigma_{cst}^o = \sigma_{ct}^o (\sigma_{st}^o)'$ is the covariance between the consumption and sustainability growth rates. The sustainability certificates price and the stock price are

$$p_t^o = \frac{u_2(c_t^o, s_t^o)}{u_1(c_t^o, s_t^o)}; \quad P_t^o = \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o (D_v(s_v^o) + p_v^o s_v^o) dv \right] \quad (53)$$

where $\xi_{t,v}^o = \xi_v^o / \xi_t^o$ is the conditional state price density.

Remark 3.3. Making explicit the dependence on an underlying production factor A in $D_t(s) = D(s, A)$ and $\Psi_t(s) = \Psi(s_t, A_t)$, and assuming A is strictly positive and evolves as an Ito process

$$\frac{dA_t}{A_t} = \mu_t^A dt + \sigma_t^A dW_t \quad (54)$$

with progressively measurable and integrable (drift) or square-integrable (volatility) coefficients, gives

$$\mu_{st}^o = -\frac{\Psi_A}{\Psi_s} \frac{A_t}{s_t^o} \mu_t^A - \frac{1}{2} \left(\frac{\Psi_{AA}}{\Psi_s} \frac{A_t^2}{s_t^o} (\sigma_t^A)^2 + 2 \frac{\Psi_{As}}{\Psi_s} A_t \sigma_t^A (\sigma_{st}^o)' + \frac{\Psi_{ss}}{\Psi_s} s_t^o (\sigma_{st}^o) (\sigma_{st}^o)' \right) \quad (55)$$

$$\sigma_{st}^o = -\frac{\Psi_A}{\Psi_s} \frac{A_t}{s_t^o} \sigma_t^A \quad (56)$$

for the coefficients of s_t^o , and

$$\mu_{ct}^o = D_A \frac{A_t}{c_t^o} \mu_t^A + D_s \frac{s_t^o}{c_t^o} \mu_{st}^o + \frac{1}{2} \left(D_{AA} \frac{A_t^2}{c_t^o} (\sigma_t^A)^2 + 2D_{As} \frac{A_t s_t^o}{c_t^o} \sigma_t^A (\sigma_{st}^o)' + D_{ss} \frac{(s_t^o)^2}{c_t^o} (\sigma_{st}^o)(\sigma_{st}^o)' \right) \quad (57)$$

$$\sigma_{ct}^o = D_A \frac{A_t}{c_t^o} (\sigma_t^A)' + D_s \frac{s_t^o}{c_t^o} (\sigma_{st}^o)' \quad (58)$$

for the coefficients of c_t^o . In these expressions $(\Psi_A, \Psi_s, \Psi_{AA}, \Psi_{As}, \Psi_{ss})$ stand for the partial derivatives of the function Ψ , and $(D_A, D_s, D_{AA}, D_{As}, D_{ss})$ for the partial derivatives of D . Partial derivatives are evaluated at equilibrium (c_t^o, s_t^o) .

The interest rate has two parts. The first part is composed of the first three terms and captures the standard relation to the moments of aggregate consumption (μ_t^c, σ_t^c) . The second part corresponds to the next three terms. It is positively related to the mean of the sustainability growth rate μ_t^s if consumption and sustainability are substitutes, negatively related to the variance σ_t^s if the product $\mathcal{E}_t^{ccs} \mathcal{E}_t^{cs}$ is positive and negatively related to the covariance between the consumption and sustainability growth rates if $\mathcal{E}_t^{ccs}$ is positive. The last two effects depend on the properties of marginal utility.

The market price of risk θ_t^o , in addition to the usual relation to the aggregate consumption growth rate, depends on the volatility of the sustainability growth rate. This term boosts the price of risk if consumption and sustainability are utility substitutes.

The price p_t of sustainability certificates is the marginal rate of substitution at the equilibrium allocation. It increases if sustainability (consumption) is more (less) desirable. The stock price is the present value of dividends, including the proceeds from the sale of sustainability certificates. Valuation uses the equilibrium stochastic discount factor, equal to the intertemporal rate of substitution between future and current consumption evaluated at the optimal allocation.

Remark 3.4. The equity risk premia, $\lambda_t = \sigma_t^P \theta_t$ in the socially optimal equilibrium with certificates and in the socially sub-optimal decentralized equilibrium with profit-maximizing firms are given by

$$\lambda_t^o = \beta_t^o A_t, \quad \text{and} \quad \lambda_t^* = \beta_t^* A_t \quad (59)$$

respectively, where

$$\beta_t^o = \left[R^c \frac{D_A}{D} - \left(R^c \frac{D_s}{D} + \mathcal{E}^{cs} \frac{1}{s} \right) \left(\frac{\Psi_A}{\Psi_s} \right) \right] (s_t^o, A_t) \sigma_{At} \sigma_t^{Po} \quad (60)$$

$$\beta_t^* = \left[R^c \frac{D_A}{D} - \left(\mathcal{E}^{cs} \frac{1}{s} \right) \left(\frac{D_{sA}}{D_{ss}} \right) \right] (s_t^*, A_t) \sigma_{At} \sigma_t^{P*}. \quad (61)$$

The equilibrium risk premium is linear in A_t in both the decentralized and the socially optimal allocations, and in neither case does a distinct sustainability factor s_t enter the risk premia expressions. However, the conditional beta is time-varying. In the social optimum, β_t depends on the sustainability choice whenever $u_{12} \neq 0$, even when sustainability has no direct effect on dividends ($D_s = 0$). If the conditional risk aversion R^c depends on s , the same mechanism applies

in the decentralized equilibrium. These results imply that empirical findings of positive ESG-alpha likely reflect misspecification of unconditional factor models that ignore the endogeneity and time variation of β_t . Conversely, empirical claims that sustainability has no effect on the risk-re trade-off within unconditional models arise from the same misspecification. Empirical asset-pricing analyses should therefore be grounded in equilibrium expressions derived from explicit assumptions about how sustainability enters dividend processes and investor preferences.

3.2 CES Preferences for Sustainability

To examine the impact of sustainability on equilibrium, we now specialize the economy. We assume uncertainty is described by a single Brownian motion W and preferences are given by the CES utility function (Gorman (1961))

$$u(c, s) = \frac{1}{1-R} (\alpha c^\rho + (1-\alpha)s^\rho)^\nu; \quad \nu = \frac{1-R}{\rho} \quad (62)$$

where ρ is the coefficient of substitution and $\epsilon = (1-\rho)^{-1}$ is the elasticity of substitution. The production set is as in the example of Section 2.5 but where productivity A follows a geometric Brownian motion with parameters (μ^A, σ^A) . In this setting s_t^o solves

$$\Psi_t(s) = \left(\frac{1-\alpha}{\alpha} \right) (A_t(1-s))^{1-\rho} + A_t(1-2s) = 0 \quad (63)$$

and equilibrium is described next.

Corollary 3.5. *Suppose profit from operations is $D(s) = As(1-s)$ where A follows a GBM process. The map $\Psi_t(\cdot)$ is strictly decreasing from $\Psi_t(0) > 0$ to $\Psi_t(1) < 0$ ensuring the existence of a unique solution $s_t^o \in [0, 1]$. Equilibrium consumption is $c_t^o = D_t(s_t^o) = A_t s_t^o (1-s_t^o)$ and the endogenous factor share is*

$$\gamma_t \equiv \frac{\alpha(c_t^o)^\rho}{\alpha(c_t^o)^\rho + (1-\alpha)(s_t^o)^\rho} \equiv \frac{\alpha(c_t^o)^\rho}{F_t} \quad (64)$$

for $t \in \mathcal{T}$. The interest rate and market prices of risk are

$$r_t = \beta + (R\gamma_t + (1-\rho)(1-\gamma_t))\mu_{ct}^o + (R - (1-\rho))(1-\gamma_t)\mu_{st}^o - \frac{1}{2} [K_{1t}(\sigma_{ct}^o)^2 + 2K_{3t}\sigma_{cs}^o + K_{2t}(\sigma_{st}^o)^2] \quad (65)$$

$$\theta_{ct}^o = (R\gamma_t + (1-\rho)(1-\gamma_t))\sigma_{ct}^o + (R - (1-\rho))(1-\gamma_t)\sigma_{st}^o \quad (66)$$

$$(67)$$

where the coefficients $K_{1t} = R_t^c P_t^c$, $K_{2t} = \mathcal{E}_t^{cs} \mathcal{E}_t^{css}$ and $K_{3t} = R_t^c \mathcal{E}_t^{ccs}$ are

$$K_{1t} = \rho^2(\nu - 1)(\nu - 2)\gamma_t^2 + 3\rho(\nu - 1)(\rho - 1)\gamma_t + (\rho - 1)(\rho - 2) \quad (68)$$

$$K_{2t} = \rho(\nu - 1)(1 - \gamma_t)[(\nu - 2)(1 - \gamma_t) + \rho - 1] \quad (69)$$

$$K_{3t} = \rho(\nu - 1)[\rho(\nu - 2)\gamma_t + \rho - 1](1 - \gamma_t) \quad (70)$$

and

$$s_t \mu_{st}^o = L_{1t} \mu_A - \frac{1}{2} (L_{2t} - 2L_{3t} - L_{4t}) \sigma_A^2 \quad (71)$$

$$s_t \sigma_{st}^o = L_{1t} \sigma_A \quad (72)$$

$$c_t^o \mu_{ct}^o = A_t [(1 - 2s_t) s_t \mu_{st}^o + s_t (1 - s_t) \mu_A] + L_{5t} A_t \sigma_A^2 \quad (73)$$

$$c_t^o \sigma_{ct}^o = [(1 - 2s_t) L_{1t} + s_t (1 - s_t)] A_t \sigma_A \quad (74)$$

with

$$L_{1t} = \frac{\rho(1 - 2s_t)(1 - s_t)}{\rho(1 - 2s_t) + 1}; \quad L_{2t} = \frac{1 - \rho}{(1 - s)^2} L_{1t}^3 \quad (75)$$

$$L_{3t} = \frac{(1 - \rho)^2(1 - 2s_t) - 2(1 - s_t)}{\rho(1 - 2s_t)(1 - s_t)} L_{1t}^2; \quad L_{4t} = (1 - \rho) L_{1t} \quad (76)$$

$$L_{5t} = -(L_{1t})^2 + (1 - 2s_t) L_{1t}. \quad (77)$$

The equilibrium certificates and stock prices are

$$p_t^o = \frac{(1 - \gamma_t) c_t^o}{\gamma_t s_t^o} \quad (78)$$

$$P_t^o = \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o c_v^o \left(1 + \frac{1 - \gamma_v}{\gamma_v} \right) dv \right] \quad (79)$$

where $y \xi_t^o = a_t F_t^\nu \gamma_t / c_t^o$, with F_t from (64), is the normalized state price density.

Remark 3.6. The equilibrium (stock) price-dividend ratio (PDR) is given by

$$PDR_t^o = \frac{1}{\gamma_0} \mathbb{E}_t \left[\int_t^\infty a_{t,v} F_{t,v}^\nu dv \right] \quad (80)$$

where $a_{t,v} = a_v / a_t$ and $F_{t,v} = F_v / F_t$. When the consumption share γ_t is low, the PDR_t^o is large. In the limit, it goes to infinity as $\gamma_t \rightarrow 0$.

Figure 4 shows equilibrium prices and allocations in the CES economy. Optimal sustainability (consumption) is greater (lower) with a certificates market, as demonstrated analytically for the general case (upper left panel). The equilibrium certificates price decreases with the elasticity of substitution $\epsilon = 1/(1 - \rho)$, reflecting the lower value of certificates when consumption becomes a better substitute for sustainability (upper right panel). In the economy with a certificates market, consumption increases and sustainability decreases (upper middle panels), whereas the interest rate

displays a U-shaped pattern and the market price of risk decreases (lower middle panels) when the elasticity of substitution increases. The interest rate increases, hence displays significantly different behavior, in the absence of a certificates market. The interest rate (market price of risk) is lower (slightly greater) in the economy with the certificates market when the elasticity of substitution exceeds a threshold (falls within a range near one). The reverse is true below this threshold (outside this range). The asset price is greater in the economy without certificates, and the return volatility is higher except in short intervals of extreme elasticity values (lower right panel). Hence, if the socially optimal equilibrium were implemented by opening the certificates market, the market price of risk would be expected to decrease, the asset price to increase, the interest rate to decrease if the elasticity of substitution is sufficiently high, and volatility to increase at intermediate levels of substitution.

Figure 5, plots the interest rate, market price of risk, asset risk premium and asset return volatility as functions of productivity in the two equilibria. First, it shows that patterns change significantly across equilibria. The interest rate, for instance, is mostly monotonic, i.e., increasing throughout or decreasing throughout, or constant with respect to productivity in the competitive equilibrium, but clearly hump-shaped up (down) in the social equilibrium with sufficiently high substitutability (complementarity). The market price of risk, the asset risk premium and its volatility are increasing, constant or very mildly hump-shaped down with respect to productivity in the competitive equilibrium, but strongly hump-shaped down (up) in the social equilibrium with sufficiently high substitutability (complementarity). Hence, there are greater variations as well as strong reversals in their sensitivities with respect to productivity in the social equilibrium. Second, there are also differences in the ordering of equilibrium outcomes as productivity and the degree of substitutability vary. The interest rate decreases except under complementarity and above a sufficiently high productivity threshold. The market price of risk decreases except under substitutability and sufficiently low productivity. The asset return volatility increases except under substitutability when productivity exceeds a low threshold. The risk premium combines the effects of volatility and the market price of risk. It increases (decreases) below (above) a productivity threshold which depends on the degree of substitutability.

Figure 6 focuses on properties of Green premia, calculated as the differences between the values of model outputs in the social equilibrium and in the decentralized equilibrium. The figure highlights fundamental effects of the transition to sustainability. It shows the Green premium component of the interest rate increases with productivity, except under substitutability and sufficiently high productivity or complementarity and sufficiently low productivity. The Green premia in the market price of risk and the asset expected return display similar patterns. They are mostly negative, except under substitutability and low productivity. They also decline with productivity except under substitutability when productivity is sufficiently high. The volatility premium in contrast, is positive hump-shaped up under complementarity and negative hump-shaped down under substitutability except below a productivity threshold where it is positive. Aside for the special case of unit elasticity, the magnitudes of Green premia vary substantially over the range of produc-

tivity values examined. Their sensitivities to productivity shocks can also display large variation across the productivity spectrum. Especially strong reversals in sensitivities can be observed under substitutability.

Overall, the introduction of a certificates market, in addition to ensuring social optimality, has a significant impact on the volatility behavior of equilibrium quantities such as the interest rate, the market price of risk and the return volatility. Although beneficial to society from a welfare point of view, financial innovation via certificates may be an important source of market fluctuations and co-fluctuations under certain circumstances.

4 Heterogeneity and Separation

We now examine the separation of ownership and control in a dynamic economy with complete markets and heterogeneous agents $i \in \mathcal{I} \equiv \{1, \dots, n\}$. Agent preferences are given by (42) and initial share endowments by $\bar{\pi}_{i0}$. We show the socially optimal choice of sustainability can be implemented by endogenous cash transfers from sustainability-sensitive agents and either a properly designed corporate governance mechanism or a traded sustainability claim.

4.1 The Economy Without a Certificates Market

Optimality conditions for agent $i \in \mathcal{I}$ in the decentralized economy without certificates market are

$$u_{i1}(c_{it}, s_t) = y_i \xi_t / a_{it} \quad (81)$$

$$\mathbb{E} \left[\int_0^\infty \xi_t c_{it} dt \right] = x_i = \bar{\pi}_{i0} P_0. \quad (82)$$

The unique solution is $c_{it} = I_i(y_i \xi_t / a_{it}, s_t)$ where $I_i(z, s_t)$ is the inverse marginal utility of consumption of the agent evaluated at the point z and y_i is the multiplier associated with the static budget constraint. Aggregating over agents shows the equilibrium state price density satisfies

$$\sum_{i \in \mathcal{I}} I_i(y_i \xi_t / a_{it}, s_t) = D_t(s_t) \quad (83)$$

which has a unique solution $\xi_t(y; s_t) = f_t(y; s_t, D_t(s_t))$ for any given sustainability level s_t and vector $y = (y_1, \dots, y_n)$. Note that $f_t(\cdot)$ is the intertemporal rate of substitution of the aggregator given aggregate consumption $C_t = D_t(s_t)$. Optimal sustainability s_t^* is determined by the firm's decision-maker via profit maximization.

Given that $\xi_t(y; s_t)$ depends on the vector of Lagrange multipliers, the budget constraint of agent i , evaluated at the (candidate) optimal consumption policy, becomes

$$\mathcal{B}_i(y_i, y; s) = \mathbb{E} \left[\int_0^\infty \xi_v(y; s_t) I_i \left(\frac{y_i \xi_t(y; s_t)}{a_{it}}, s_t \right) dv \right] - \bar{\pi}_{i0} P_0(y) = 0 \quad (84)$$

for $i \in \mathcal{I}$. To proceed, taking account of the optimal firm choice s_t^* such that $D_t'(s_t^*) = 0$, we assume

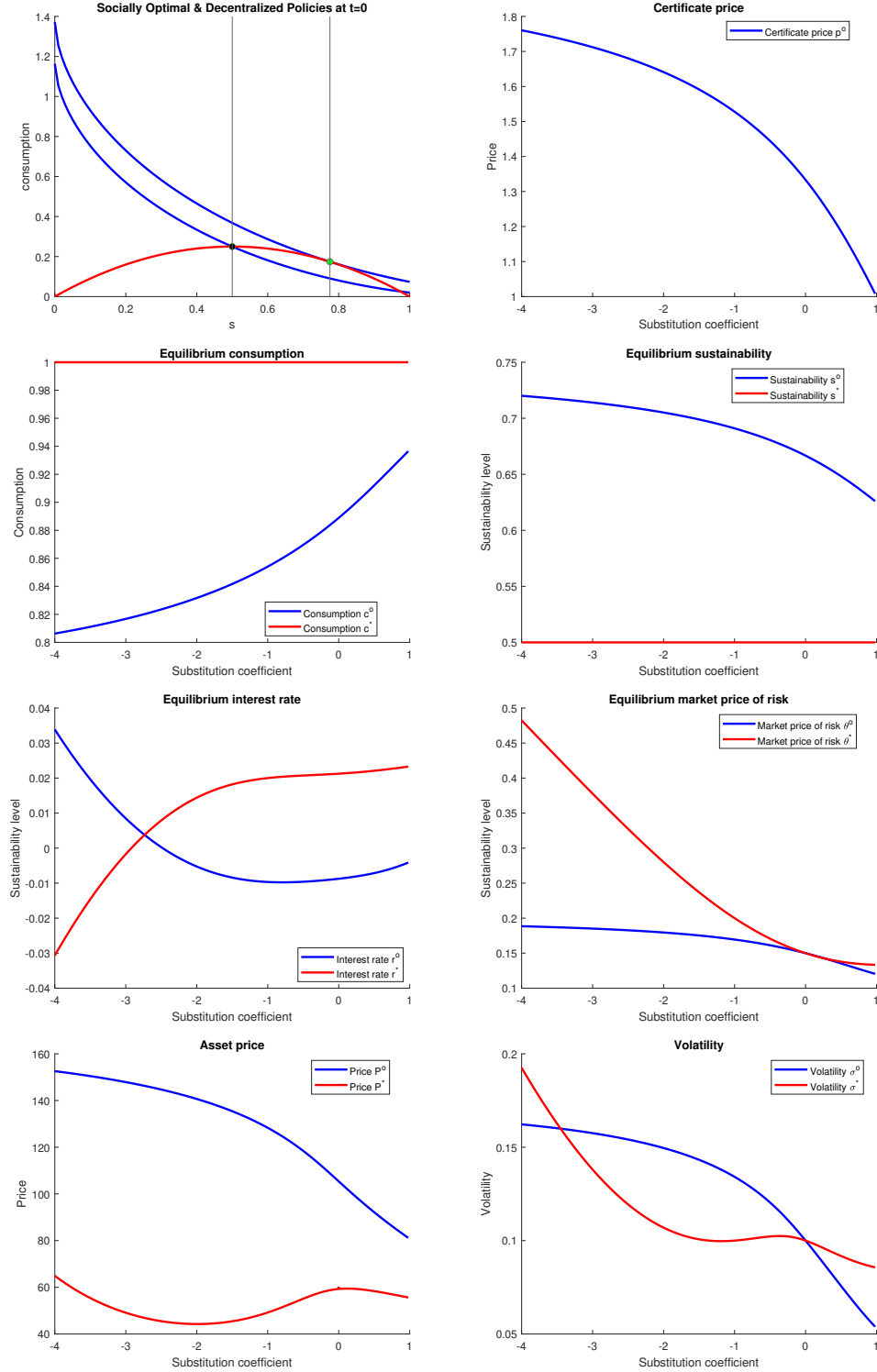


Figure 4: Panels show equilibria (upper left), the certificate price (upper right), consumption (upper middle left), sustainability (upper middle right), interest rate (lower middle left), market price of risk (lower middle right), asset price (lower left) and return volatility (lower right) as functions of the elasticity parameter ρ in the dynamic model with CES preferences at time $t = 0$. Socially optimal (decentralized) equilibrium in blue (red). Productivity follows a geometric Brownian motion with $\mu_A = 0.02$ and $\sigma_A = 0.25$, and $A_0 = 4$. Other parameters are $\beta = 0.01$, $R = 2$, $\alpha = 0.5$.

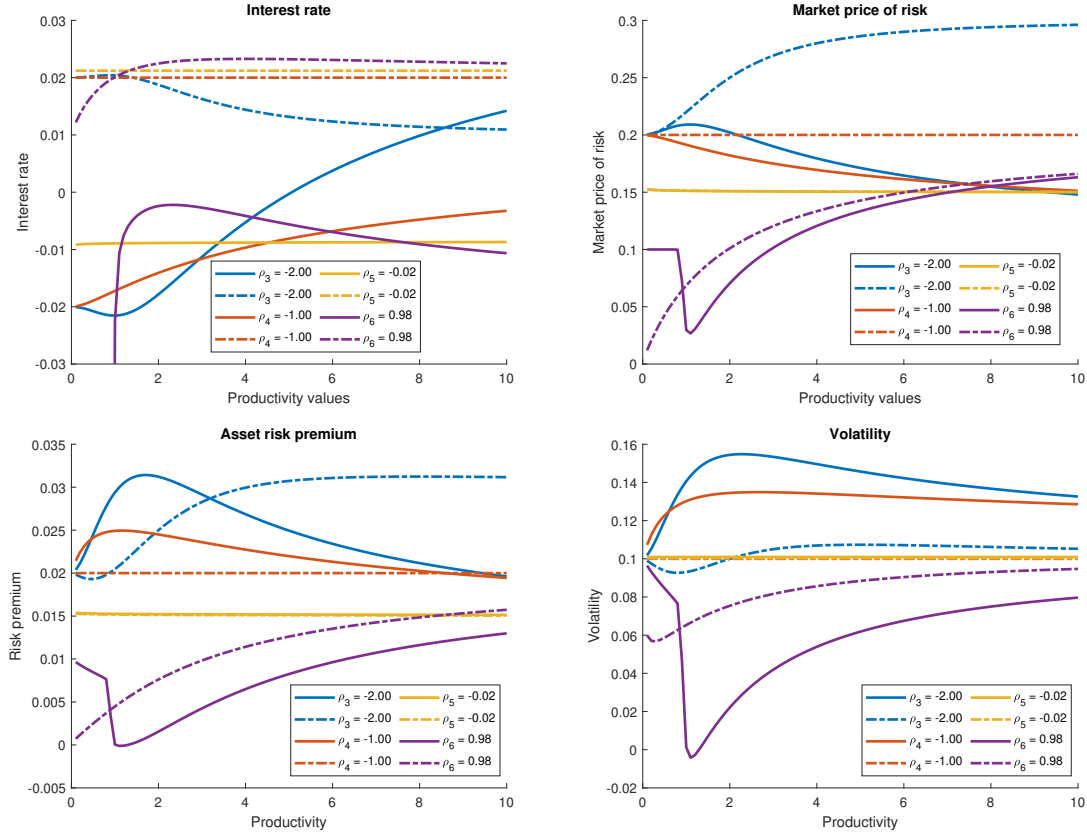


Figure 5: Panels show the interest rate (upper left), market price of risk (upper right), asset risk premium (lower left) and asset return volatility (lower right) as functions of productivity A in the dynamic model with CES preferences at time $t = 0$. Socially optimal (decentralized) equilibrium is displayed as solid (dashed) curves. Elasticity of substitution indicated in legend boxes. Productivity follows a geometric Brownian motion with $\mu_A = 0.02$ and $\sigma_A = 0.25$. Other parameters are $\beta = 0.01$, $R = 2$, $\alpha = 0.5$.

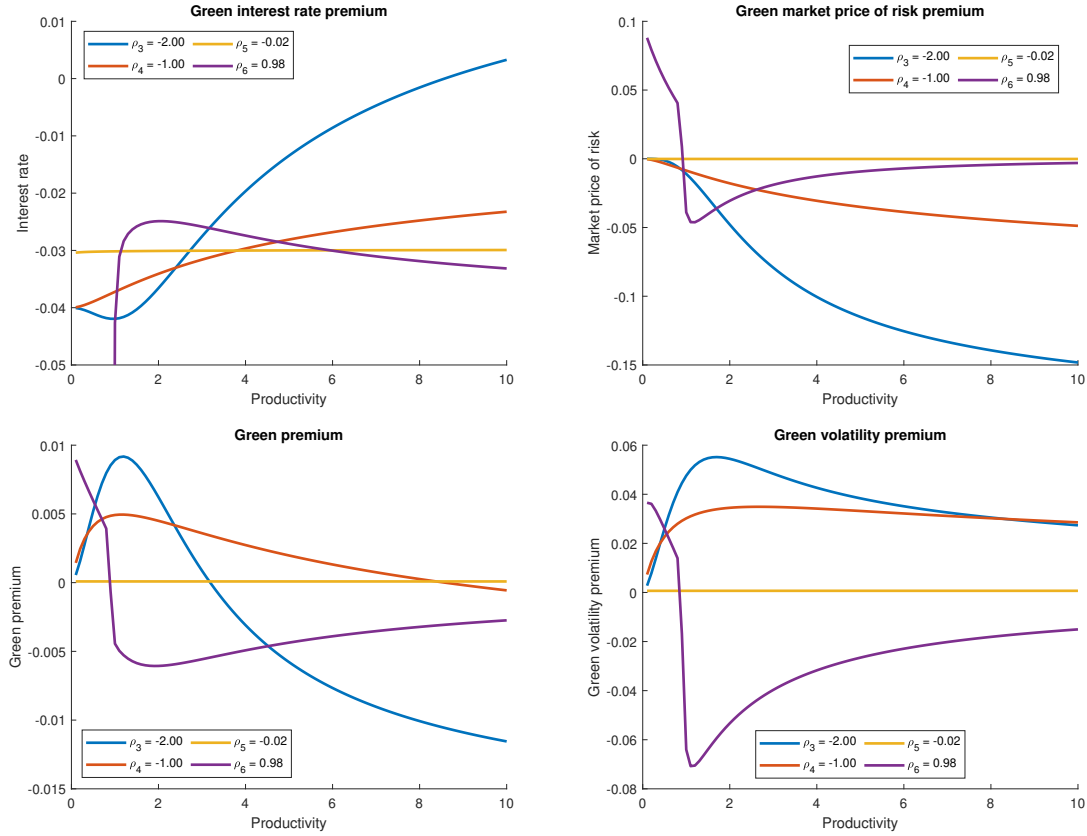


Figure 6: Panels show the Green interest rate premium (upper left), Green market price of risk premium (upper right), Green asset return premium (lower left) and Green volatility premium (lower right) as functions of productivity A in the dynamic model with CES preferences at time $t = 0$. All premia are calculated as the differences between their values in the socially optimal equilibrium and the decentralized equilibrium. Elasticity of substitution value indicated in legend boxes. Productivity follows a geometric Brownian motion with $\mu_A = 0.02$ and $\sigma_A = 0.25$. Other parameters are $\beta = 0.01$, $R = 2$, $\alpha = 0.5$.

Assumption 4.1. Let $\mathcal{B}(y; s^*)$ denote the vector of budget constraints evaluated at the optimal firm policy s^* . The system of equations $\mathcal{B}(y; s^*) = 0$ has a solution y^* .

Proposition 4.2. Suppose Assumption 4.1 holds with solution y^* . A competitive rational expectations equilibrium exists. Optimal sustainability is $s_t^* = [D_t']^{-1}(0)$. Optimal consumption is $c_{it}^* = I_i(y_i \xi_t^* / a_{it}, s_t^*)$ where the state price density is given by $\xi_t^* = f(y^*; s_t^*, D_t(s_t^*))$ and the function $f(\cdot)$ solves (83). Aggregate consumption $c_t^* = D_t^*(s_t^*)$ evolves according to

$$\frac{dc_t^*}{c_t^*} = \mu_t^c dt + \sigma_t^c dW_t \equiv \mu_t^D(s_t^*) dt + \sigma_t^D(s_t^*) dW_t. \quad (85)$$

Given the aggregator's utility function $u(c_t, s_t)$, the equilibrium state price density is $y \xi_t = u_1(D_t(s_t^*), s_t^*)$ for $t \in [0, \infty)$ with $y = u_1(D_0(s_0^*), s_0^*)$. The equilibrium interest rate and market price of risk are

$$r_t = \beta_t + R_t^c \mu_t^c - \frac{1}{2} R_t^c P_t^c (\sigma_t^c)^2 + \mathcal{E}_t^{cs} \mu_t^s - \frac{1}{2} \mathcal{E}_t^{cs} \mathcal{E}_t^{css} (\sigma_t^s)^2 - R_t^c \mathcal{E}_t^{ccs} \sigma_t^{cs} \quad (86)$$

$$\theta_t^c = R_t^c (\sigma_t^c)' + \mathcal{E}_t^{cs} (\sigma_t^s)' \quad (87)$$

where all preference coefficients relate to the aggregator and are evaluated at the equilibrium $(D(s^*), s^*)$. The equilibrium stock price is

$$P_t = \mathbb{E}_t \left[\int_0^\infty \xi_{t,v} D_v(s_v^*) dv \right] = \mathbb{E}_t \left[\int_0^\infty \frac{u_1(D_v(s_v^*), s_v^*)}{u_1(D_t(s_t^*), s_t^*)} D_v(s_v^*) dv \right] \quad (88)$$

where the conditional state price density $\xi_{t,v} = \xi_v / \xi_t$ is the marginal rate of substitution of the aggregator.

4.2 The Planning Problem

Consider now the planner problem

$$\max_{(c_i, s)} \sum_{i \in \mathcal{I}} \lambda_{it} a_{it} u_i(c_{it}, s_t) dt \quad \text{subject to} \quad \sum_i c_{it} = D_t(s_t) \quad (89)$$

where λ_{it} are Negishi utility weights. Without loss of generality, we can set the subjective discount factors $a_{it} = 1$ for $i \in \mathcal{I}$ by rescaling the weights. Optimization subject to the resource constraint leads to the first order conditions

$$\lambda_{it} u_{i1}(c_{it}, s_t) = \gamma_t \quad (90)$$

$$\sum_{i \in \mathcal{I}} \lambda_{it} u_{i2}(c_{it}, s_t) = -\gamma_t D_t'(s_t) \quad (91)$$

$$\sum_{i \in \mathcal{I}} c_{it} = D_t(s_t). \quad (92)$$

where γ_t is the Lagrange multiplier for the resource constraint. Solving the first equation gives $c_{it} = I_i(\gamma_t/\lambda_{it}, s_t)$. Dividing the second equation by γ_t , using that γ_t does not depend on i to bring it inside the summation, and simplifying shows that s_t solves

$$\Psi_t(s_t; \gamma_t) \equiv \sum_{i \in \mathcal{I}} \frac{u_{i2} \left(I_i \left(\frac{\gamma_t}{\lambda_{it}}, s_t \right), s_t \right)}{u_{i1} \left(I_i \left(\frac{\gamma_t}{\lambda_{it}}, s_t \right), s_t \right)} + D'_t(s_t) = 0. \quad (93)$$

and since $u_{i1}(I_i(y, s), s) = y$, the denominator simplifies and

$$\Psi_t(s_t; \gamma_t) \equiv \sum_{i \in \mathcal{I}} \frac{\lambda_{it}}{\gamma_t} u_{i2} \left(I_i \left(\frac{\gamma_t}{\lambda_{it}}, s_t \right), s_t \right) + D'_t(s_t) = 0. \quad (94)$$

Following the approach in previous sections, we assume

Assumption 4.3. *The stochastic map $\Psi_t(s_t; \gamma_t)$ has a unique zero, i.e., there exists a unique $\mathfrak{s}_t(\gamma_t)$ such that $\Psi_t(\mathfrak{s}_t(\gamma_t); \gamma_t) = 0$, for each $t \in [0, \infty)$.*

The multiplier for the resource constraint then satisfies

$$\mathcal{R}_t(\gamma_t) \equiv \sum_i I_i \left(\frac{\gamma_t}{\lambda_{it}}, \mathfrak{s}_t(\gamma_t) \right) - D_t(\mathfrak{s}_t(\gamma_t)) = 0. \quad (95)$$

Assumption 4.4. *The stochastic map $\mathcal{R}_t(\gamma_t)$ has a zero, i.e., there exists γ_t^o such that $\mathcal{R}_t(\gamma_t^o) = 0$, for each $t \in [0, \infty)$.*

The next result establishes the existence of a social optimum.

Proposition 4.5. *Suppose Assumptions 4.3 and 4.4 hold and let γ_t^o be a solution of $\mathcal{R}_t(\gamma) = 0$ for $t \in [0, \infty)$. The socially optimal policy is (c_{it}^o, s_t^o) where $c_{it}^o = I_i(\gamma_t^o/\lambda_{it}, s_t^o)$ and $s_t^o = \mathfrak{s}_t(\gamma_t^o)$ is the unique solution of the equation $\Psi_t(s_t; \gamma_t^o) = 0$, for $t \in [0, \infty)$. The socially optimal sustainability index process exceeds the corresponding optimal index in the decentralized economy: $s_t^o > s_t^*$ and $c_t^o > c_t^*$ for all $t \in [0, \infty)$. The Fisher separation between ownership and control fails.*

Proof of Proposition 4.5. Existence of the socially optimal allocation follows directly from Assumptions 4.3 and 4.4. The existence of the allocation c_{it} follows from the construction of optimal consumption policies. To complete the proof, we note from the second optimality condition above that $s_t^* < s_t^o$ and that $D_t(s^*) > D_t(s_t^o)$. ♦

Remark 4.6. *The social optimum depends on the vector of Negishi weights $s_t^o = s_t^o(\lambda)$. Different weights on agent utilities induce different levels of socially optimal sustainability. But irrespective of the weights $s_t^o(\lambda) > s_t^*$ for $t \in [0, \infty)$.*

4.3 The Economy with Sustainability Transfers

Consider now an economy with a population of diverse agents with sustainability preferences and a firm. Agents can influence the sustainability choice of the firm by contributing cash (transfers)

to support its operations. They maximize preferences to determine consumption demands and supplies of transfers to the firm. They hire the firm with the mandate to maximize total profits consisting of profits from operations and from aggregate transfers. They also set up a governance board to verify transfers and monitor compliance with the mandate. The governance board uses a near unanimity voting rule, specifically “unanimity minus one”, as a decision mechanism. We assume there aren’t any other frictions, i.e., the financial market is complete and information is perfect.

Agent $i \in \mathcal{I}$ demands a level of sustainability based on their willingness to fund the firm.³ If $\tau_{it}(s)$ is the transfer supply, the optimization problem of agent $i \in \mathcal{I}$ becomes

$$\begin{aligned} \max_{c_{it}, s_t} \mathbb{E} \left[\int_0^\infty a_{it} u_i(c_{it}, s_t) \right] \quad \text{s.t.} \\ \mathbb{E} \left[\int_0^\infty \xi_t(c_{it} + \tau_{it}(s_t)) dv \right] \leq \bar{\pi}_{i0} P_0 \end{aligned}$$

leading to the first order conditions

$$u_{i1}(c_{it}, s_t) = y_i \xi_t / a_{it} \quad (96)$$

$$u_{i2}(c_{it}, s_t) = (y_i \xi_t / a_{it}) \tau'_{it}(s_t) \quad (97)$$

$$\mathbb{E} \left[\int_0^\infty \xi_t(c_{it} + \tau_{it}(s_t)) dt \right] = x_i = \bar{\pi}_{i0} P_0. \quad (98)$$

From the first condition, we obtain the consumption demand $c_{it} = I_i(y_i \xi_t / a_{it}, s_t)$ where $I_i(z, s)$ is the inverse marginal utility of agent i at the point z . From the second condition, $\tau'_{it}(s_t) = m_{it}(c_{it}, s_t)$ for each $i \in \mathcal{I}$, where m_{it} is the marginal rate of substitution. Substituting optimal consumption and integrating with respect to s gives the transfer supply of agent i

$$\tau_{it}(s; \xi_t, y_i) = \int_0^s m_{it} \left(I_i \left(\frac{y_i \xi_t}{a_{it}}, z \right), z \right) dz. \quad (99)$$

To obtain this expression, we used the initial condition $\tau_{it}(0; \xi_t, y_i) = 0$, implied by agent i ’s budget constraint (given prices, it is suboptimal to add any positive random variable independent of s to the transfer supply because it would reduce consumption, hence welfare. Negative random variables would be rejected by the governance board). Summing over agents gives the aggregate transfer supply

$$\tau_t(s; \xi_t, y) = \sum_{i \in \mathcal{I}} \tau_{it}(s; \xi_t, y_i) = \int_0^s \sum_{i \in \mathcal{I}} m_{it} \left(I_i \left(\frac{y_i \xi_t}{a_{it}}, z \right), z \right) dz \quad (100)$$

where $y = (y_1, \dots, y_n)$ is the vector of Lagrange multipliers for the agents budget constraints. The supply function depends on the level of sustainability and is also parameterized by the SPD ξ .

³Equivalently, they choose a transfer amount to support a level of sustainability.

The mandate of the firm is to maximize total profits consisting of profits from operations $D_t(s)$ and profits from transfers $\tau_t(s; \xi_t, y)$. At each time $t \in [0, \infty)$ it solves

$$\max_s \Pi_t(s; \xi_t, y) \equiv D_t(s) + \tau_t(s; \xi_t, y), \quad (101)$$

where $\tau_t(s; \xi_t, y)$ is defined in (100). The necessary condition for optimal sustainability is

$$\Pi'_t(s; \xi_t, y) = D'(s_t) + \tau'_t(s_t; \xi_t, y) = 0 \quad (102)$$

for $t \in [0, \infty)$, where the derivative is with respect to the first argument. Since $s \in [0, 1]$, an optimal choice exists and is given by $s_t = H_t(0; \xi_t, y)$ where $H(\cdot; \xi_t, y)$ is the inverse marginal profit function. To proceed, redefine the marginal profit function as

$$\Psi_t(s; \xi_t, y) = \sum_{i \in \mathcal{I}} m_{it} \left(I_i \left(\frac{y_i \xi_t}{a_{it}}, s \right), s \right) + D'_t(s) \quad (103)$$

and note that the firm's choice of sustainability satisfies $\Psi_t(s; \xi_t, y) = 0$. Also, note that the solution depends on ξ_t .

Assumption 4.7. *The stochastic map $\Psi_t(s; \xi_t, y)$ has an interior zero corresponding to a profit maximum, i.e., there exists $\mathfrak{s}_t(\xi_t, y) \in (0, 1)$ such that $\Psi_t(\mathfrak{s}_t(\xi_t, y); \xi_t, y) = 0$, for each $t \in [0, \infty)$ and $\mathfrak{s}_t(\xi_t, y) \in (0, 1)$ maximizes profits.*

Assumption 4.7 is a weak requirement since optimization is over a compact set. The assumption is automatically satisfied if the marginal profit function satisfies the limit condition $\lim_{s \rightarrow 1} \Psi(s; \xi_t, y) < 0$. (note that $\lim_{s \rightarrow 0} \Psi(s; \xi_t, y) = \infty$).

Market clearing in the goods market now requires

$$\Phi_t(\xi_t; y) \equiv \sum_{i \in \mathcal{I}} I_i \left(\frac{y_i \xi_t}{a_{it}}, \mathfrak{s}_t(\xi_t, y) \right) - D_t(\mathfrak{s}_t(\xi_t, y)) = 0. \quad (104)$$

Assumption 4.8. *Equation (104) has a solution $\xi_t = f_t(y)$.*

The function $f_t(y)$ identifies the structure of the equilibrium SPD as a function of the vector of Lagrange multipliers for the agents' budget constraints. Evaluating optimal policies at that function gives $s_t(y) = \mathfrak{s}_t(f(y), y)$ for sustainability and $c_{it}(y_i, y) = I_i(y_i f_t(y)/a_{it}, s_t(y))$ for individual i 's consumption. Agents expenditures functions and the stock price become

$$\mathcal{B}_i(y_i, y) = \mathbb{E} \left[\int_0^\infty f_v(y) (c_{iv}(y_i, y) + \tau_{iv}(s_v(y))) dv \right]; \quad i \in \mathcal{I} \quad (105)$$

$$P_0(y) = \mathbb{E} \left[\int_0^\infty f_v(y) \Pi_v(y) dv \right] \quad (106)$$

for $i \in \mathcal{I}$. We then assume

Assumption 4.9. Let $\mathcal{B}(y)$ denote the vector of expenditure functions. The system of equations $\mathcal{B}(y) = x_i = \bar{\pi}_i P_0(y)$ has a solution y^o .

In light of Assumption 4.9, the previous Assumption 4.7 could be weakened by only requiring that it hold at the solution y^o .

With all these elements in place, we can now state the following

Proposition 4.10. Consider the economy with heterogeneous agents and sustainability transfer flows described above, and suppose Assumptions 4.7-4.9 hold. The competitive equilibrium with sustainability transfers exists. The equilibrium SPD is $\xi_t^o = f(y^o)$ where y^o satisfies the budget constraints of agents. Equilibrium sustainability is $s_t^o = \mathfrak{s}_t(f(y^o), y^o)$. Equilibrium agent consumption and aggregate consumption are respectively $c_{it}(y_i^o, y^o)$ for $i \in \mathcal{I}$ and $c_t^o = D(s_t^o)$. The stock price $P_t^o = P_t(y^o)$ is the present value of profits. Profit maximization is unanimously agreed upon and socially optimal. The Fisher separation between ownership and control holds.

Remark 4.11. (i) Note that agents can inject funds in the firm on their own volition, but cannot decide to withdraw funds for personal benefit. Any mandate to take a transfer supply scheme into account for firm management must be supported by all shareholders minus one. Any withdrawal to the benefit of a single agent would therefore be opposed by the governance board.

(ii) The equilibrium sustainability in the economy with sustainability transfers is greater than in the economy without, $s_t^o > s_t^*$. Value maximization is universally agreed upon as the objective of the firm.

4.4 Implementation of the Transfer Equilibrium

Implementation of the transfer equilibrium described above can proceed as follows: taking as given the equilibrium state price density process $\xi^o = f(y^o)$ where y^o satisfies the vector of budget constraints

1. Shareholders (agents) optimize and transmit transfer supply functions to the firm along with a mandate to maximize total profits.
2. A governance board composed of shareholders is set up to verify transfer functions and implementation of the mandate.
3. The firm optimizes to determine $s_t^o = \mathfrak{s}_t(\xi_t^o, y^o)$ and the optimal profits $\Pi^o(s_t^o)$ to be distributed as dividends over time.
4. Agents send the present values of their respective contributions evaluated at equilibrium prices to the firm at time 0.

The mechanism described above has two main advantages. First, the governance board verifies that the transfer supplies claimed do not incorporate any payment or rebate that is not incentive compatible. Second, once the value of equilibrium transfers is paid, agents cannot renege and free ride on the outcome.

Remark 4.12. *The first order condition for sustainability choice of the social planner's problem is essentially the Samuelson efficiency condition for producing a public good. The implementation of the social optimum through transfers by maintaining the mandate for profit maximization is essentially analogue to a Lindahl equilibrium for the production of a public good. The caveats for the practicality of the mechanism are also the same: the mechanism requires full information about the MRSs between consumption and sustainability of agents. An alternative Vickery-Clarke-Groves-type truthful preference-revelation mechanism is not a viable solution here, as these mechanisms while enforcing truthful revelation of willingness to pay for sustainability are not budget balanced.*

Implementation of the mechanism is also possible through financial innovation. Consumers, in addition to initial shares of stock $\bar{\pi}_{i0}$, hold a short position ψ_{i0} in a sustainability-linked asset with equilibrium price

$$V_t = \mathbb{E} \left[\int_t^\infty \xi_{t,v} \tau_v(s_v^o) dv \right].$$

where

$$\psi_{i0} = - \frac{\mathbb{E} \left[\int_0^\infty \xi_v \tau_{iv}(s_v^o) dv \right]}{\mathbb{E} \left[\int_0^\infty \xi_v \tau_v(s_v^o) dv \right]}.$$

Initial wealth is therefore $X_{i0} = \bar{\pi}_{i0}P_0 + \psi_{i0}V_0$. The static budget constraint for consumption is then

$$\mathbb{E} \left[\int_0^\infty \xi_t c_{it} dt \right] \leq \bar{\pi}_{i0}P_0 + \psi_{i0}V_0.$$

If $D_t(s_t^o)$ depends on two sources of risk, say W^D and W^s (e.g., profitability and cost risks), then the sustainability-linked asset can also help to span the market, and it becomes a non-redundant traded asset. Here, the stock is a claim on total dividends $D_t(s_t^o) + \tau_t(s_t^o)$. Alternatively, if the stock only gives rights to operational cash flows D_t and the sustainability-linked asset gives rights to transfer cash flows τ_t , then the stock price is

$$\tilde{P}_t = \mathbb{E}_t \left[\int_t^\infty \xi_{t,v} D_v(s_v^o) dv \right].$$

Initial wealth is then a position in the stock and a position in the sustainability-linked transfer asset V_t given by

$$\tilde{\psi}_{i0} = \bar{\pi}_{i0} + \psi_{i0}.$$

This position can be positive or negative depending on the agents willingness to pay for sustainability and the initial shares of the stock. The budget constraint of the consumer is then

$$\mathbb{E} \left[\int_0^\infty \xi_t c_{it} dt \right] \leq \bar{\pi}_{i0}\tilde{P}_0 + \tilde{\psi}_{i0}V_0.$$

The implementation of the mechanism will only require that consumers be willing to hold the sustainability-linked transfer asset. They are willing to do so as these positions are optimal from an individual standpoint.

Remark 4.13. *The creation of the sustainability-linked asset requires complete information about the agents' willingness to substitute consumption for sustainability. An alternative to financial innovation here is for the Government to tax every consumer by $\psi_{i0}V_0$ and to subsidize the producer by V_0 contingent on the firm maximizing total profits. As $\sum_i \psi_{i0} = 1$ such a scheme is self-financing. The informational requirement for the government to set up this scheme is that it must know the willingness to pay of each consumer. The question therefore is whether it is better for shareholders, represented by the governance board, to implement the governance mechanism through financial innovation or for the government to process individualized taxes to finance the sustainability subsidies to firms and at the same time enforce that firms maximize total profits.*

Remark 4.14. *The central planning problem characterizes all Pareto Optimal equilibria. In the absence of sustainability transfers, the financial market equilibrium with profit-maximizing firms is not Pareto optimal. By the second welfare theorem, any Pareto efficient allocation of resources can be achieved through a competitive market by first using lump-sum transfers to redistribute initial endowments, and then allowing the free market to operate. This is achieved in the economy with sustainability transfers. However, sustainability itself is a public good. As noted in Remark 4.12, the first-order condition of the planner's problem is essentially the Samuelson (1954) condition for the provision of a public good, and the equilibrium with sustainability transfers is essentially a Lindahl (1958) equilibrium for public goods. A governance board is needed to prevent free-riding.*

4.5 Asset Pricing in the Transfer Equilibrium

In order to solve for the interest rate and the market price of risk, it is convenient to express the state price density in terms of the preferences of the aggregator $y\xi_t = u(c_t^o, s_t^o)$. Risk aversion and other preference coefficients of the aggregator can be extracted from the aggregate demand function $I(y\xi_t, s_t^o) = \sum_{i \in \mathcal{I}} I_{it}(y_i\xi_t, s_t^o)$. In equilibrium, the aggregate demand function is equal to the supply of the consumption good.

Proposition 4.15. *Consider the economy with a sustainability transfers and suppose Assumptions 4.7-4.9 hold. Let equilibrium sustainability and consumption evolve according to*

$$\frac{ds_t^o}{s_t^o} = \mu_{st}^o dt + \sigma_{st}^o dW_t; \quad \frac{dc_t^o}{c_t^o} = \mu_{ct}^o dt + \sigma_{ct}^o dW_t. \quad (107)$$

The equilibrium interest rate and market price of risk are

$$r_t^o = \beta_t + R_t^c \mu_{ct}^o - \frac{1}{2} R_t^c P_t^c (\sigma_{ct}^o)^2 + \mathcal{E}_t^{cs} \mu_{st}^o - \frac{1}{2} \mathcal{E}_t^{cs} \mathcal{E}_t^{css} (\sigma_{st}^o)^2 - R_t^c \mathcal{E}_t^{ccs} \sigma_{cst}^o \quad (108)$$

$$\theta_t^o = R_t^c (\sigma_{ct}^o)' + \mathcal{E}_t^{cs} (\sigma_{st}^o)' \quad (109)$$

where aggregate preference coefficients can be expressed in terms of the consumption shares $\omega_{it} =$

c_{it}^o/c_t^o as

$$R_t^c = \left(\sum_i \omega_{it} \frac{1}{R_{it}^c} \right)^{-1} ; \quad \mathcal{E}_t^{cs} = \sum_i \omega_{it} \frac{R_t^c}{R_{it}^c} \mathcal{E}_{it}^{cs} \quad (110)$$

$$P_t^c = \sum_i \omega_{it} \left(\frac{R_t^c}{R_{it}^c} \right)^2 P_{it}^c \quad (111)$$

$$\mathcal{E}_t^{ccs} = \frac{P_t^c}{R_t^c} \mathcal{E}_t^{cs} - R_t^c \sum_i \omega_{it} \left(\frac{P_{it}^c \mathcal{E}_{it}^{cs}}{(R_{it}^c)^2} + \frac{\mathcal{E}_{it}^{ccs}}{R_{it}^c} \right) \quad (112)$$

$$\begin{aligned} \mathcal{E}_t^{css} = & \frac{P_t^c}{R_t^c} \mathcal{E}_t^{cs} - 2R_t^c \sum_i \omega_{it} \left(\frac{P_{it}^c \mathcal{E}_{it}^{cs}}{(R_{it}^c)^2} + \frac{\mathcal{E}_{it}^{ccs}}{R_{it}^c} \right) \\ & + \sum_i \omega_{it} \left(\frac{R_t^c}{R_{it}^c} \frac{P_{it}^c (\mathcal{E}_{it}^{cs})^2}{\mathcal{E}_{it}^{cs} R_{it}^c} + \frac{R_t^c \mathcal{E}_{it}^{cs}}{R_{it}^c \mathcal{E}_{it}^{cs}} (-2\mathcal{E}_{it}^{ccs} + \mathcal{E}_{it}^{css}) \right) \end{aligned} \quad (113)$$

and are all evaluated at the equilibrium allocation (c^o, s^o) . Also, $\sigma_{cst}^o = \sigma_{ct}^o (\sigma_{st}^o)'$ is the covariance between the consumption and sustainability growth rates while $\sigma_{ct}^o = \sigma_{ct}^o (\sigma_{ct}^o)'$ and $\sigma_{st}^o = \sigma_{st}^o (\sigma_{st}^o)'$ are the variances of the consumption and sustainability growth rates, respectively. The sustainability claim price and the stock price are

$$V_t^o = \mathbb{E}_t \left[\int_0^\infty \xi_{t,v}^o \tau(s_v^o) dv \right] \quad (114)$$

$$P_t^o = \mathbb{E}_t \left[\int_0^\infty \xi_{t,v}^o (D_v(s_v^o) + \tau(s_v^o)) dv \right] \quad (115)$$

where $\xi_{t,v}^o = \xi_v^o / \xi_t^o$ is the conditional state price density.

The interest rate and the market price of risk have the same structures as in the economy with a representative agent. However, the preference coefficients entering these relations are aggregates of individual preference coefficients, hence reflect the diversity among agents, in particular their different attitudes with respect to sustainability. Properties of equilibrium are nevertheless the same as long as preference aggregates have the same properties as in the representative agent model.

4.6 Further Perspectives

Friedman's paradigm of shareholder value maximization, which directs managers to pursue profit maximization, is theoretically attractive because, under the assumption that consumers possess strictly monotonic utility over private consumption, it yields a socially optimal allocation and is straightforward to implement. Hart and Zingales (2017), however, observe that shareholders are not exclusively motivated by pecuniary returns but also by social, environmental, and ethical considerations. On this basis, they propose that firms ought to maximize shareholder welfare rather than financial value alone, and suggest shareholder voting as a mechanism to incorporate such non-pecuniary objectives into corporate decision-making.

We demonstrate that strict profit maximization can generate socially suboptimal equilibria once

preferences are extended to encompass social, environmental, and ethical dimensions. To remedy this inefficiency, we develop mechanisms that enable managers to maximize total dividends, defined as the sum of conventional dividend payments and aggregate shareholder transfer payments earmarked for social objectives. Furthermore, we illustrate how these transfers can be securitized into financial instruments explicitly linked to social, environmental, and ethical outcomes. The principal challenge arises from the public good nature of these outcomes, together with the informational requirements necessary to elicit and aggregate heterogeneous shareholder preferences in order to design optimal transfers.

The proposed transfer-payment mechanism circumvents the minority disenfranchisement inherent in the Hart and Zingales voting scheme: each shareholder contributes an individualized transfer consistent with the social optimum. In this way, the framework preserves the conceptual integrity of the profit-maximization paradigm while mitigating the risks of corporate mission creep associated with democratic shareholder processes or self-regulation. Implementation nonetheless remains demanding. It necessitates the establishment of a shareholder consortium capable of aggregating preferences and enforcing contribution rules, thereby deterring free riding through underfunding of the public good. The introduction of outcome-linked assets provides a partial solution by aligning incentives and attenuating the tendency toward free riding.

5 Conclusion

In this paper we examined the impact of preferences for sustainability on the competitive equilibria of dynamic and static stock market economies with production. We first showed that the Fisher separation principle between ownership and control fails when agents care about sustainability and that the resulting competitive equilibrium is socially sub-optimal. Second, we showed that opening a competitive market in sustainability certificates restores the social optimality of the competitive equilibrium. Third, we provided conditions for the existence of Green or Brown premia in stocks or bonds and showed that both exhibit Green premia if consumption and sustainability are utility complement. Finally, we documented the impact of sustainability on the equilibrium interest rate, the price of consumption risk, the certificates price and the stock price in dynamic single and multiple agents economies with certificates. Both the interest rate and the price of consumption risk inherit new components tied to the moments of the socially optimal sustainability level. In a calibrated single agent economy with CES utility function, the interest rate, market price of risk and return volatility can become more volatile, exhibit significant volatility changes and reversals and display important co-variation changes depending on the elasticity of substitution upon the opening of the certificates market.

Our analysis provides new insights about the consequences of a transition to a sustainable economy. It first shows that desirable socially optimal outcomes may come at the expense of production. Altering manufacturing processes may indeed reduce efficiency and financial profitability, hence the consumption of traditional goods and the satisfaction derived from it. But it also raises

the consumption of sustainability, an important aspect of welfare for an increasingly significant constituency. Second, it highlights the complexity of implications for financial markets. It shows that the interest rate, the market price of risk and the asset return volatility may react in different directions depending on the degree of substitution between traditional goods and sustainability. For economies that view sustainability as a close substitute for consumption, they all fall upon transitioning, except when productivity is very low where the price of risk and volatility may increase. Perhaps more significantly, their sensitivities to productivity shocks could increase or change sign. Volatilities of equilibrium quantities and/or variations in such volatilities could therefore become by-products of the transition, under certain circumstances. Fluctuations, however, are not a detriment to society in this case since they are associated with a welfare improvement. Third, the analysis demonstrates that the transition to a sustainable economy can be achieved through a spectrum of regulatory, market-based, and governance mechanisms. Government interventions such as Pigouvian taxes remain one option, while market-based instruments—including certificate systems and, critically, the design of sustainability-linked financial assets—offer decentralized ways to attain socially optimal outcomes. Corporate governance arrangements that support monitoring and voluntary contributions further complement these approaches. Choosing among mechanisms requires weighing their respective efficiency, informational and administrative costs, and distributional consequences. Ultimately, a judicious combination of regulatory tools, financial innovation, and governance structures is likely to provide the most effective pathway for implementing the sustainability transition.

While the analysis provides a foundation for understanding the transition to a sustainable economic system, it abstracts from two important dimensions. First, we take production technologies as given and therefore do not consider how the pursuit of sustainability may induce innovation, create new sectors, or reshape long-run productivity growth. Second, we omit the role of labor—both in the production process and in the adjustment of the workforce to emerging technologies—despite the fact that skill formation and retraining are central to a real-world sustainability transition. A fuller account of these innovation and labor-market dynamics lies beyond the scope of the present model but represents a promising direction for future research.

Our results demonstrate that profit maximization and standard NPV rules remain socially optimal in the presence of pro-social preferences provided that firms maximize the joint value of production revenues and sustainability-linked assets that securitize the proceeds of socially optimal transfer payments implemented through certificate schemes. We further show that, owing to the public-goods nature of sustainability, the effective deployment of such market-based mechanisms necessitates modifications to corporate governance structures to ensure credible monitoring, enforcement, and alignment between private incentives and social objectives.

6 Appendix: Single Agent Diffusion Economy

The economy is as in Section 3, with the additional proviso that productivity follows the diffusion process

$$dA_t = A_t (\mu_A(A_t)dt + \sigma_A(A_t)dW_t). \quad (116)$$

The socially optimal sustainability level solves $\Psi(s_t^o, A_t) = 0$ where

$$\Psi(s, A) = \frac{u_2(D(s, A), s)}{u_1(D(s, A), s)} + D_s(s, A) \quad (117)$$

whereas the profit maximizing sustainability level solves $D_s(s_t^*, A_t) = 0$ with $D_s = \partial D / \partial s$. We now derive explicit expressions for equilibrium quantities in the socially optimal equilibrium (central planner and equilibrium with certificates) and in the socially inefficient financial market equilibrium with profit-maximizing firms.

6.1 Socially optimal equilibrium

The next proposition describes the SPD, market price of risk, stock price, and stock return volatility in the socially optimal equilibrium.

Lemma 6.1. *The socially optimal level of sustainability satisfies*

$$ds_t^o = s_t^o (\mu_s(s_t^o, A_t)dt + \sigma_s(s_t^o, A_t)dW_t) \quad (118)$$

where the drift and volatility functions

$$s\mu_s(s, A) = \left[-A \frac{\Psi_A}{\Psi_s} \mu_A - \frac{A^2}{2} \left\{ \Psi_A^2 \frac{\Psi_{ss}}{\Psi_s^3} - \Psi_A \left(\frac{\Psi_{sA} + \Psi_{As}}{\Psi_s^2} \right) + \frac{\Psi_{AA}}{\Psi_s} \right\} \|\sigma_A\|^2 \right] (s, A) \quad (119)$$

$$s\sigma^s(s, A) = \left[-A \frac{\Psi_A}{\Psi_s} \sigma_A \right] (s, A) \quad (120)$$

are evaluated at (s_t^o, A_t) .

Proof of Lemma 6.1 Using Ito's Lemma and the optimality condition $\Psi(s_t, A_t) = 0$ leads to,

$$\Psi_s ds_t + \Psi_A dA_t + \frac{1}{2} \{ \Psi_{ss} d[s]_t + (\Psi_{sA} + \Psi_{As}) d[s, A]_t + \Psi_{AA} d[A]_t \} = 0$$

where all functions are evaluated at (s_t^o, A_t) , and therefore

$$\Psi_s d[s, A]_t = -\Psi_A d[A]_t; \quad \Psi_s d[s]_t = -\Psi_A d[s, A]_t = (\Psi_A^2 / \Psi_s) d[A]_t \quad (121)$$

$$ds_t = -\frac{\Psi_A}{\Psi_s} dA_t - \frac{1}{2} \left\{ \Psi_A^2 \frac{\Psi_{ss}}{\Psi_s^3} - \Psi_A \left(\frac{\Psi_{sA} + \Psi_{As}}{\Psi_s^2} \right) + \frac{\Psi_{AA}}{\Psi_s} \right\} d[A]_t. \quad (122)$$

Identifying the drift and volatility coefficients in (118) with those in (119)-(120) establishes the claim. ♦

Next we show that the diffusion coefficients above can be found in explicit form. Define $x := D(s, A)$. For brevity, write the partial derivatives of u as (u_1, u_2) all evaluated at (x, s) , and likewise for the partials $(D_s, D_A, D_{ss}, D_{sA}, D_{AA})$ of D . The marginal rate of substitution $m(x, s) \equiv u_2(x, s)/u_1(x, s)$ has derivatives

$$\begin{aligned} m_x &= \frac{u_{21}u_1 - u_2u_{11}}{u_1^2}; & m_s &= \frac{u_{22}u_1 - u_2u_{12}}{u_1^2} \\ m_{xx} &= \frac{u_{211}u_1^2 - u_2u_1u_{111} - 2u_{21}u_1u_{11} + 2u_2(u_{11})^2}{u_1^3} \\ m_{ss} &= \frac{u_{222}u_1^2 - u_2u_1u_{122} - 2u_{22}u_1u_{12} + 2u_2(u_{12})^2}{u_1^3} \\ m_{xs} &= \frac{(u_{221}u_1 + u_{21}u_{12} - u_{22}u_{11} - u_2u_{112})u_1 - 2(u_{21}u_1 - u_2u_{11})u_{12}}{u_1^3}. \end{aligned}$$

With this notation, we obtain

Lemma 6.2. *The coefficients of the socially optimal equilibrium are*

$$\Psi_s = m_x D_s + m_s + D_{ss}; \quad \Psi_A = m_x D_A + D_{sA} \quad (123)$$

$$\Psi_{ss} = m_{xx}(D_s)^2 + 2m_{xs}D_s + m_x D_{ss} + m_{ss} + D_{sss}, \quad (124)$$

$$\Psi_{sA} = m_{xx}D_s D_A + m_x D_{sA} + m_{xs}D_A + D_{ssa}, \quad (125)$$

$$\Psi_{AA} = m_{xx}(D_A)^2 + m_x D_{AA} + D_{sAA}. \quad (126)$$

The socially optimal dividend $D_t^o = D(s_t^o, A_t)$ satisfies

$$dD_t^o = D_t^o (\mu_D(s_t^o, A_t)dt + \sigma_D(s_t^o, A_t)dW_t) \quad (127)$$

where the drift and volatility functions

$$\begin{aligned} \mu_D(s, A) &= \left[\frac{D_s \mu_s + D_A \mu_A + \frac{1}{2} \{ D_{ss} \|\sigma_s\|^2 + (D_{sA} + D_{As}) \sigma_s \cdot \sigma_A + D_{AA} \|\sigma_A\|^2 \}}{D} \right] (s, A) \\ \sigma_D(s, A) &= \left[\frac{D_s s \sigma_s + D_A A \sigma_A}{D} \right] (s, A) \end{aligned} \quad (128)$$

are evaluated at (s_t^o, A_t) .

Theorem 6.3. *Suppose that $m(0, 1) < \infty$. Under this condition, the equation $\Psi(s, A) = 0$ has a unique zero, i.e., there exists a unique social optimum s^o .*

Proof of Theorem 6.3. Since $D'(0, A) = +\infty$ and $D'(1, A) = -\infty$, the map $\Psi(s, A)$ has the

limits

$$\Psi(0, A) = m(0, A) + D_s(0, A) = +\infty \quad (129)$$

$$\Psi(1, A) = m(0, A) + D_s(1, A) = -\infty. \quad (130)$$

By continuity of $\Psi(s, A)$ with respect to s , the equation $\Psi(s, A) = 0$ has a solution. Uniqueness follows from the fact that the composition $u(D(s, A), s)$ is strictly concave with respect to s , which implies that the numerator of

$$\Psi_s(s, A) = \frac{u_2(D(s, A), s) + D_s(s, A)u_1(D(s, A), s)}{u_1(D(s, A), s)} \quad (131)$$

is strictly decreasing by assumption. ♦

Remark 6.4. Note that the condition $m(0, 1) < +\infty$ is satisfied if $u_2(0, 1) < \infty$, which is a weak requirement. Also note that a sufficient condition for the strict concavity of the composition map $u(D(s, A), s)$ is $u_{11}D_s^2 + 2u_{12}D_s + u_{22} < 0$, which is always satisfied since the quadratic equation $u_{11}z^2 + 2u_{12}z + u_{22} = 0$ in z does not have a real solution (the determinant $\Delta = u_{11}u_{22} - u_{12}^2 > 0$ by concavity of $u(c, s)$ over the domain $\mathcal{C} \times \mathcal{S}$), implying the left hand side of this equation is negative.

Proposition 6.5. Consider the diffusion economy and assume that $\Psi(s_t, A_t) = 0$ has a unique solution s_t^o for all $t \in \mathcal{T}$. The socially optimal SPD is

$$\xi_t^o = \frac{a_t u_1(D(s_t^o, A_t), s_t^o)}{a_0 u_1(D(s_0^o, A_0), s_0^o)}. \quad (132)$$

With relative risk aversion $R^c(s, A) = -u_{11}(D(s, A), s)D(s, A)/u_1(D(s, A), s)$ and elasticity of marginal utility $\mathcal{E}^{cs}(s, A) = -u_{12}(D(s, A), s)s/u_1(D(s, A), s)$, the socially optimal market price of risk is

$$\theta_t^o = [R^c \sigma_D + \mathcal{E}^{cs} \sigma_s](s_t^o, A_t) \quad (133)$$

where

$$\sigma_D(s, A) = \left[\frac{D_s s \sigma_s + D_A A \sigma_A}{D} \right](s, A); \quad \sigma_s(s, A) s = \left[-\frac{\Psi_A}{\Psi_s} A \sigma_A \right](s, A). \quad (134)$$

$$\frac{\Psi_A}{\Psi_s} = \frac{\left(\frac{u_{21}u_1 - u_2u_{11}}{u_1^2} \right) D_A + D_{sA}}{\left(\frac{u_{21}u_1 - u_2u_{11}}{u_1^2} \right) D_s + \frac{u_{22}u_1 - u_2u_{12}}{u_1^2} + D_{ss}}. \quad (135)$$

The socially optimal stock price is

$$P_t^o = \mathbb{E} \left[\int_t^T \xi_{t,v}^o (D(s_v^o, A_v) - D_s(s_v^o, A_v)s_v^o) \right] \quad (136)$$

where $\xi_{t,v}^o = \xi_v^o / \xi_t^o$ is the conditional state price density and the socially optimal stock return volatility is

$$\sigma_t^{P,o} = \theta_t^o + \sigma_{1t}^{P,o} - \sigma_{2t}^{P,o} - \sigma_{3t}^{P,o} \quad (137)$$

with components

$$P_t \sigma_{1t}^{P,o} = \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o ([D_A - D_{sA}s](s_v^o, A_v) \mathcal{D}_t A_v - D_{ss}(s_v^o, A_v) s_v^o \mathcal{D}_t s_v^o) \right] \quad (138)$$

$$P_t \sigma_{2t}^{P,o} = \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o [D - D_{ss}](s_v^o, A_v) \left(\left[R^c \frac{D_s}{D} + \frac{\mathcal{E}^{cs}}{s} \right] (s_v^o, A_v) \mathcal{D}_t s_v^o \right) dv \right] \quad (139)$$

$$P_t \sigma_{3t}^{P,o} = \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o [D - D_{ss}](s_v^o, A_v) \left(\left[R^c \frac{D_A}{D} \right] (s_v^o, A_v) \mathcal{D}_t A_v \right) dv \right] \quad (140)$$

where the Malliavin derivative $\mathcal{D}_t s_v^o$ is given by

$$\mathcal{D}_t s_v^o = - \left[\frac{\Psi_A}{\Psi_s} \right] (s_v^o, A_v) \mathcal{D}_t A_v \quad (141)$$

and $\mathcal{D}_t A_v$ solves the linear SDE

$$d\mathcal{D}_t A_v = (\partial_A (A \mu_A(A)) dv + \partial_A (A \sigma_A(A)) dW_v) \mathcal{D}_t A_v \quad (142)$$

subject to the initial condition $\mathcal{D}_t A_t = \sigma_A(A_t)$. In this expression, $\partial_{A \cdot}$ is the partial derivative relative to A , and all the derivatives are evaluated at A_v .

Next, we consider the additive separable case $u(c, s) = v(c) + w(s)$. In this instance, the marginal rate of substitution $m(x, s) \equiv u_2(s)/u_1(x)$ where $u_2(s) = w'(s)$ and $u_1(c) = v'(c)$, has derivatives

$$\begin{aligned} m_x &= \frac{-u_2 u_{11}}{u_1^2}; & m_s &= \frac{u_{22} u_1}{u_1^2}; & m_{xx} &= \frac{-u_2 u_1 u_{111} + 2u_2 (u_{11})^2}{u_1^3} \\ m_{ss} &= \frac{u_{222} u_1^2}{u_1^3}; & m_{xs} &= \frac{-u_{22} u_{11} u_1}{u_1^3}. \end{aligned}$$

Our next theorem establishes existence and uniqueness of the social optimum.

Theorem 6.6. *Consider the additive separable case $u(c, s) = v(c) + w(s)$. There exists a unique solution s_t^o to $\Psi(s, A) = 0$. The optimal sustainability and the associated SPD satisfy*

$$\frac{u_2(s_t^o)}{u_1(D(s_t^o, A))} + D_s(s_t^o, A) = 0; \quad \xi_t^o = \frac{a_t u_1(D(s_t^o, A_t))}{a_0 u_1(D(s_0^o, A_0))}. \quad (143)$$

The market price of risk is $\theta_t^o = [R^c \sigma_D](s_t^o, A_t)$ where σ_D is as in Proposition 6.5. The volatility

coefficient is $\sigma_t^{P,o} = \theta_t^o + \sigma_{1t}^{P,o} - \sigma_{2t}^{P,o} - \sigma_{3t}^{P,o}$ with $(\sigma_{1t}^{P,o}, \sigma_{3t}^{P,o})$ as in the Proposition and

$$P_t \sigma_{2t}^{P,o} = \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o [D - D_s s] (s_v^o, A_v) \left(\left[R^c \frac{D_s}{D} \right] (s_v^o, A_v) \mathcal{D}_t s_v^o \right) dv \right]. \quad (144)$$

The Malliavin derivatives $(\mathcal{D}_t s_v^o, \mathcal{D}_t A_v)$ satisfy (141)-(142), but with coefficients (μ_s, σ_s) of (118) specialized to the economy under consideration.

Proof of Theorem 6.6. Since $u_{12} = 0$, the composite utility $u(D(s, A), s) = v(D(s, A)) + w(s)$ is concave with respect to s . The marginal rate of substitution also satisfies $m(0, 1) = 0 < +\infty$. The existence of a unique root follows from Theorem 6.3. The remaining expressions can be derived by specializing the formulas in Proposition 6.5. ♦

Remark 6.7. Since profit maximization entails $D(s^*, A) = 0$, Theorem 6.6 shows that sustainability matters for firm decision-making even if the utility function is additively separable. The only case where it fails to matter is when preferences do not depend on it, i.e., when the agent is purely financially-motivated.

6.2 Profit Maximizing Competitive Equilibrium

Proposition 6.8. The SPD in the competitive financial market equilibrium with profit maximizing firms is

$$\xi_t^* = \frac{a_t u_1(D(s_t^*, A_t), s_t^*)}{a_0 u_1(D(s_0^*, A_0), s_0^*)} \quad (145)$$

The equilibrium market price of risk is

$$\theta_t^* = [R^c \sigma^D + R^s \sigma^s] (s_t^*, A_t) \quad (146)$$

where

$$\sigma^D(s, A) = \left[\frac{D_A A \sigma^A}{D} \right] (s, A); \quad \sigma^s(s, A) s = \left[-\frac{D_{sA}}{D_{ss}} A \sigma^A \right] (s, A). \quad (147)$$

The stock price is

$$P_t^* = \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^* D(s_v^*, A_v) \right] \quad (148)$$

The stock price volatility coefficient is

$$\sigma_t^{P,*} = \theta_t^* + \sigma_{1t}^{P,*} - \sigma_{2t}^{P,*} - \sigma_{3t}^{P,*} \quad (149)$$

with components

$$P_t^* \sigma_{1t}^{P,*} = \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^* D_A(s_v^*, A_v) \mathcal{D}_t A_v \right] \quad (150)$$

$$P_t^* \sigma_{2t}^{P,*} \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^* D(s_v^*, A_v) \left(\left[\mathcal{E}^{cs} \frac{1}{s} \right] (s_v^*, A_v) \mathcal{D}_t s_v^* \right) dv \right] \quad (151)$$

$$P_t^* \sigma_{3t}^{P,*} = \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^* D(s_v^*, A_v) \left[R^c \frac{D_A}{D} \right] (s_v^*, A_v) \mathcal{D}_t A_v dv \right] \quad (152)$$

where

$$\mathcal{D}_t s_v^* = - \left[\frac{D_{sA}}{D_{ss}} \right] (s_v^*, A_v) \mathcal{D}_t A_v \quad (153)$$

and the Malliavin derivative $\mathcal{D}_t A_v$ solves the linear SDE (142) subject to the same initial condition.

Proof of Proposition 6.8 For the profit maximizing competitive equilibrium

$$\begin{aligned} \Psi_s &= D_{ss}; & \Psi_A &= D_{sA} \\ \Psi_{ss} &= D_{sss}; & \Psi_{sA} &= D_{ssA}; & \Psi_{AA} &= D_{sAA}. \end{aligned}$$

Furthermore $D_s(s_t^*, A_t) = 0$. The expressions announced then follow by using the same steps as in the proof of the socially optimal equilibrium. $\blacklozenge$

6.3 CES Utility with Outer Curvature $1 - R$

6.3.1 Model

We consider the Gorman (1961) generalized CES specification which allows for an outer curvature parameter R

$$u(c, s) = \frac{\left(\alpha c^\rho + (1 - \alpha) s^\rho \right)^{(1-R)/\rho}}{1 - R}, \quad \alpha \in (0, 1), \quad \rho \neq 0.$$

This nests standard CES utility when $R = 0$ and introduces an additional curvature (risk aversion) when $R \neq 0$. Let $x \equiv D(s, A)$ and define $\Psi(s, A)$ as in (117) with partial derivatives of D denoted by $D_s, D_A, D_{ss}, D_{sA}, D_{AA}, \dots$. We also assume productivity A follows a geometric Brownian motion.

6.3.2 Equilibrium

Ratio of marginal utilities. Differentiating $u(c, s)$ gives

$$u_1 = (\alpha c^\rho + (1 - \alpha) s^\rho)^{(1-R-\rho)/\rho} \alpha c^{\rho-1} \quad (154)$$

$$u_2 = (\alpha c^\rho + (1 - \alpha) s^\rho)^{(1-R-\rho)/\rho} (1 - \alpha) s^{\rho-1}. \quad (155)$$

so that

$$m(c, s) = \frac{u_2(c, s)}{u_1(c, s)} = \frac{1 - \alpha}{\alpha} \left(\frac{s}{c} \right)^{\rho-1} \equiv K \left(\frac{s}{c} \right)^r; \quad r \equiv \rho - 1; \quad K = \frac{1 - \alpha}{\alpha}.$$

Useful derivatives of $m(x, s) = K s^r x^{-r}$.

$$m_x = -\frac{r}{x}m, \quad m_s = \frac{r}{s}m,$$

$$m_{xx} = \frac{r(r+1)}{x^2}m, \quad m_{ss} = \frac{r(r-1)}{s^2}m, \quad m_{xs} = -\frac{r^2}{sx}m.$$

First derivatives of Ψ .

$$\Psi_s = m_x D_s + m_s + D_{ss} = \left(-\frac{r}{x}D_s + \frac{r}{s}\right)m + D_{ss},$$

$$\Psi_A = m_x D_A + D_{sA} = -\frac{rm}{x}D_A + D_{sA}.$$

Second derivatives of Ψ .

$$\begin{aligned} \Psi_{ss} &= m_{xx}(D_s)^2 + 2m_{xs}D_s + m_x D_{ss} + m_{ss} + D_{sss} \\ &= \left(\frac{r(r+1)}{x^2}(D_s)^2 - \frac{2r^2}{sx}D_s - \frac{r}{x}D_{ss} + \frac{r(r-1)}{s^2}\right)m + D_{sss}, \\ \Psi_{sA} &= m_{xx}D_s D_A + m_x D_{sA} + m_{xs}D_A + D_{ssA} \\ &= \left(\frac{r(r+1)}{x^2}D_s D_A - \frac{r}{x}D_{sA} - \frac{r^2}{sx}D_A\right)m + D_{ssA}, \\ \Psi_{AA} &= m_{xx}(D_A)^2 + m_x D_{AA} + D_{sAA} = \left(\frac{r(r+1)}{x^2}(D_A)^2 - \frac{r}{x}D_{AA}\right)m + D_{sAA}. \end{aligned}$$

Marginal rate of substitution. With $x = D(s, A)$,

$$m(x, s) = K \left(\frac{s}{x}\right)^r = \frac{1-\alpha}{\alpha} \left(\frac{s}{x}\right)^{\rho-1}.$$

Cobb–Douglas limit: as $\rho \rightarrow 0$ (so $r \rightarrow -1$), we obtain

$$m = \frac{1-\alpha}{\alpha} \frac{x}{s},$$

recovering the Cobb–Douglas case. The outer curvature parameter R does not affect these ratios, but it will influence second-order welfare comparisons and dynamic optimization through the intertemporal aggregator.

Technology specification With $D = As(1-s)$, the coefficients appearing in the equilibrium solution simplify as follows

(a) Preference coefficients

$$\begin{aligned}
m &= KA^{-r}(1-s)^{-r}; \quad r = \rho - 1 = -\epsilon^{-1} \\
R^c &= \frac{1}{F} (\rho(1-\nu) + 1 - \rho)\alpha c^\rho + (1-\rho)(1-\alpha)s^\rho = R\gamma + (1-\rho)(1-\gamma) \\
\mathcal{E}^{cs} &= \frac{1}{F} \rho(1-\nu)(1-\alpha)s^\rho = (\rho + R - 1)(1-\gamma); \quad \gamma = \frac{\alpha c^\rho}{F}
\end{aligned}$$

(b) Technology derivatives

$$\begin{aligned}
D_s &= A(1-2s); \quad D_A = s(1-s); \quad D_{sA} = 1-2s \\
D_{ss} &= -2A; \quad D_{sss} = 0; \quad D_{ssa} = -2 \quad D_{AA} = 0 \\
D_A - D_{sA}s &= s^2; \quad D - D_s s = As^2 \\
\frac{D_s}{D} &= \frac{1-2s}{s(1-s)}; \quad \frac{D_A}{D} = \frac{1}{A}; \quad \frac{D_{sA}}{D} = \frac{1-2s}{As(1-s)}
\end{aligned}$$

(c) Equilibrium map derivatives:

$$\begin{aligned}
\Psi(s) &= K(A(1-s))^{-r} + A(1-2s) = 0 \\
\Psi_s &= r \left(-\frac{D_s}{D} + \frac{1}{s} \right) m - 2A = rKA^{-r}(1-s)^{-r-1} - 2A = -A \frac{1+\rho(1-2s)}{1-s} \\
\Psi_A &= -rm \frac{D_A}{D} + D_{sA} = -rKA^{-r-1}(1-s)^{-r} + 1-2s = \rho(1-2s) \\
\Psi_{ss} &= \left(r(r+1) \left(\frac{D_s}{D} \right)^2 - \frac{2r^2}{s} \frac{D_s}{D} - r \frac{D_{ss}}{D} + \frac{r(r-1)}{s^2} \right) m \\
&= - \left(r(r+1) \left(\frac{1-2s}{s(1-s)} \right)^2 - \frac{2r^2}{s} \frac{1-2s}{s(1-s)} - r \frac{-2}{s(1-s)} + \frac{r(r-1)}{s^2} \right) A(1-2s) \\
&= - \frac{r(r+1)A(1-2s)}{(1-s)^2} \\
\Psi_{sA} &= \left(r(r+1) \frac{D_s}{D} \frac{D_A}{D} - r \frac{D_{sA}}{D} - \frac{r^2}{s} \frac{D_A}{D} \right) m - 2 \\
&= - \left(r(r+1) \frac{1-2s}{s(1-s)} - r \frac{1-2s}{s(1-s)} - \frac{r^2}{s} \right) (1-2s) - 2 = r^2 \left(\frac{1-2s}{1-s} \right) - 2 \\
\Psi_{AA} &= \left(r(r+1) \left(\frac{D_A}{D} \right)^2 \right) m = \rho(1-\rho) \frac{1-2s}{A}
\end{aligned}$$

(d) Equilibrium sustainability process drift and volatility

$$\begin{aligned}
s\mu_s(s) &= \frac{\rho(1-2s)(1-s)}{1+\rho(1-2s)}\mu^A - \frac{1}{2}M(s)(\sigma^A)^2 \equiv L_1(s)\mu^A - \frac{1}{2}M(s)(\sigma^A)^2 \\
M(s) &= \left(\frac{\rho(1-2s)(1-s)}{(1+\rho(1-2s))} \right)^3 \frac{(-r)}{(1-s)^2} \\
&\quad - 2 \left(\frac{\rho(1-2s)(1-s)}{(1+\rho(1-2s))} \right)^2 \left(\frac{r^2(1-2s)-2(1-s)}{\rho(1-2s)(1-s)} \right) - \frac{\rho(1-\rho)(1-2s)(1-s)}{(1+\rho(1-2s))} \\
&\equiv L_1(s)^3 \frac{-r}{(1-s)^2} - 2L_1(s)^2 \left(\frac{r^2(1-2s)-2(1-s)}{\rho(1-2s)(1-s)} \right) - L_1(s)(1-\rho) \\
s\sigma_s(s) &= -\frac{\Psi_A}{\Psi_s}A_t\sigma^A = \frac{\rho(1-2s)(1-s)}{1+\rho(1-2s)}\sigma^A \equiv L_1(s)\sigma^A
\end{aligned}$$

(e) Volatility in the social optimum

$$\begin{aligned}
D_v^o &= A_v s_v^o(1-s_v^o); \quad L_{1v}^o \equiv L_1(s_v^o) \\
\mathcal{D}_t A_v &= \sigma^A A_v; \quad \mathcal{D}_t s_v^o = -(\Psi_A/\Psi_s)\sigma^A A_v = L_{1v}^o \sigma^A \\
R_t^{c,o} &= (1/F_t^o) (R\alpha(D_t^o)^\rho + (1-\rho)(1-\alpha)(s_t^o)^\rho) \\
\mathcal{E}_t^{cs,o} &= (1/F_t^o)\rho(1-\nu)(1-\alpha)(s_t^o)^\rho \\
F_t^o &= \alpha(D_t^o)^\rho + (1-\alpha)(s_t^o)^\rho \\
y^o \xi_t^o &= (F_t^o)^{\nu-1} \alpha(D_t^o)^{\rho-1} = (F_t^o)^\nu \gamma_t^o / D_t^o \\
P_t \sigma_{1t}^{P,o} &= \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o ((s_v^o)^2 \mathcal{D}_t A_v + 2A_v s_v^o \mathcal{D}_t s_v^o) dv \right] \\
&= \sigma^A \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o A_v ((s_v^o)^2 + 2s_v^o L_{1v}^o) dv \right] \\
P_t \sigma_{2t}^{P,o} &= \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o A_v (s_v^o)^2 \left(\left[R^c \frac{D_s}{D} + \frac{\mathcal{E}^{cs}}{s} \right] (s_v^o, A_v) \mathcal{D}_t s_v^o \right) dv \right] \\
&= \sigma^A \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o A_v (s_v^o)^2 \left(\left[R^c \frac{D_s}{D} + \frac{\mathcal{E}^{cs}}{s} \right] (s_v^o, A_v) \right) L_{1v}^o dv \right] \\
P_t \sigma_{3t}^{P,o} &= \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o A_v (s_v^o)^2 \left([R^c] (s_v^o, A_v) \frac{\mathcal{D}_t A_v}{A_v} \right) dv \right] \\
&= \sigma^A \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^o A_v (s_v^o)^2 R_v^{c,o} dv \right]
\end{aligned}$$

(f) Volatility in the decentralized equilibrium without certificates

$$\begin{aligned}
s_t^* &= 1/2; & \mathcal{D}_t s_v^* &= 0; & \mathcal{D}_t A_v &= \sigma^A A_v; & D(s_v^*, A_v) &= A_v/4 \\
R_t^{c,*} &= \frac{1}{F_t^*} \left(R\alpha \left(\frac{A_t}{4} \right)^\rho + (1-\rho)(1-\alpha) \left(\frac{1}{2} \right)^\rho \right) \\
F_t^* &= \alpha \left(\frac{A_t}{4} \right)^\rho + (1-\alpha) \left(\frac{1}{2} \right)^\rho \\
y^* \xi_t^* &= (F_t^*)^{\nu-1} \alpha \left(\frac{A_t}{4} \right)^{\rho-1} = (F_t^*)^\nu \frac{\gamma_t^*}{D_t^*} \\
P_t^* \sigma_{1t}^{P,*} &= \frac{\sigma^A}{4} \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^* A_v \right]; & P_t^* \sigma_{2t}^{P,*} &= 0 \\
P_t^* \sigma_{3t}^{P,*} &= \frac{\sigma^A}{4} \mathbb{E}_t \left[\int_t^\infty \xi_{t,v}^* R_v^{c,*} A_v dv \right]
\end{aligned}$$

6.3.3 Alternative volatility formulas

The price formula (79), under CES preferences, leads to the discounted stock price

$$y^o \xi_t^o P_t^o = \mathbb{E}_t \left[\int_t^\infty y^o \xi_v^o c_v^o \left(\frac{1}{\gamma_v} \right) dv \right] = \mathbb{E}_t \left[\int_t^\infty a_v (F_v^o)^\nu dv \right] \quad (156)$$

where we used $y^o \xi_v^o = a_v (F_v^o)^\nu \gamma_v^o / c_v^o$ to obtain the right hand side. Applying the rules of Malliavin calculus shows that

$$y^o (\mathcal{D}_t \xi_t^o) P_t^o + y^o \xi_t^o \mathcal{D}_t P_t^o = \mathbb{E}_t \left[\int_t^\infty a_v \mathcal{D}_t (F_v^o)^\nu dv \right] \quad (157)$$

where

$$\begin{aligned}
\mathcal{D}_t \xi_t^o &= -\xi_t^o (\theta_t^o)'; & \mathcal{D}_t P_t^o &= P_t^o \sigma_t^o \\
\mathcal{D}_t (F_v^o)^\nu &= \nu (F_v^o)^{\nu-1} \mathcal{D}_t F_v^o = \nu (F_v^o)^\nu (\mathcal{D}_t F_v^o / F_v^o) \\
\mathcal{D}_t F_v^o &= \rho [\alpha (D_v^o)^{\rho-1} \mathcal{D}_t D_v^o + (1-\alpha) (s_v^o)^{\rho-1} \mathcal{D}_t s_v^o] \\
\mathcal{D}_t s_v^o &= -(\Psi_A / \Psi_s) \sigma^A A_v = L_{1v} \sigma_A \\
\mathcal{D}_t D_v^o &= A_v s_v^o (1 - s_v^o) \sigma_A + A_v (1 - 2s_v^o) \mathcal{D}_t s_v^o
\end{aligned}$$

and

$$\begin{aligned}
\frac{\mathcal{D}_t D_v^o}{D_v^o} &= \left[\sigma_A + \frac{1 - 2s_v^o}{1 - s_v^o} \frac{\mathcal{D}_t s_v^o}{s_v^o} \right] = \left[1 + \frac{\rho(1 - 2s_v^o)^2}{s_v^o(1 + \rho(1 - 2s_v^o))} \right] \sigma_A \\
\frac{\mathcal{D}_t F_v^o}{F_v^o} &= \rho \left[\gamma_v^o \frac{\mathcal{D}_t D_v^o}{D_v^o} + (1 - \gamma_v^o) \frac{\mathcal{D}_t s_v^o}{s_v^o} \right] \\
&= \rho \left[\gamma_v^o + \left(\gamma_v^o \frac{1 - 2s_v^o}{1 - s_v^o} + (1 - \gamma_v^o) \right) \frac{\rho(1 - 2s_v^o)(1 - s_v^o)}{s_v^o(1 + \rho(1 - 2s_v^o))} \right] \sigma_A \\
&= \rho \left[\gamma_v^o + (-\gamma_v^o s_v^o + 1 - s_v^o) \frac{\rho(1 - 2s_v^o)}{s_v^o(1 + \rho(1 - 2s_v^o))} \right] \sigma_A \\
&= \rho \left[\frac{\gamma_v^o s_v^o + (1 - s_v^o)\rho(1 - 2s_v^o)}{s_v^o(1 + \rho(1 - 2s_v^o))} \right] \sigma_A \equiv \rho M_v^o \sigma_A.
\end{aligned}$$

Volatility can then be written as

$$\sigma_v^o = \theta_t^o + \nu \rho \frac{c_t^o}{\gamma_t^o P_t^o} \mathbb{E}_t \left[\int_t^\infty a_{t,v}(F_{t,v}^o)^\nu M_v^o dv \right] \sigma_A \quad (158)$$

for the social equilibrium. Similarly for the competitive equilibrium

$$y^* \xi_t^* P_t^* = \mathbb{E}_t \left[\int_t^\infty y^* \xi_v^* c_v^* dv \right] = \mathbb{E}_t \left[\int_t^\infty a_v(F_v^*)^{\nu-1} \alpha(c_v^*)^\rho dv \right] \quad (159)$$

where we used $y^o \xi_v^* = a_v(F_v^*)^\nu \gamma_v^* / c_v^* = a_v(F_v^*)^{\nu-1} \alpha(c_v^*)^{\rho-1}$ to obtain the right hand side. Following the same steps as above with $\mathcal{D}_t s_v^* = 0$ and $\mathcal{D}_t D_v^* = D_v^* \sigma_A$ gives $\mathcal{D}_t F_v^o / F_v^o = \rho \gamma_v^o \sigma_A$ and

$$\sigma_v^* = \theta_t^* + \rho \frac{c_t^*}{\gamma_t^* P_t^*} \mathbb{E}_t \left[\int_t^\infty a_{t,v}(F_{t,v}^*)^\nu \gamma_v^* ((\nu - 1)\gamma_v^* + 1) dv \right] \sigma_A \quad (160)$$

for volatility in the competitive equilibrium.

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